

Chapter 1.1 - Real Numbers

Integers: counting numbers, zero, and the negative of the counting numbers.

$$\text{ex: } \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Rational Numbers: quotients of two integers with a nonzero denominator; terminating or repeating decimals.

$$\text{ex: } 5, 2.7, -\frac{3}{4}, \frac{1}{2}, \frac{1}{3} \dots$$

Irrational Numbers: real numbers that are not rational numbers; nonterminating and nonrepeating decimals.

$$\text{ex: } \sqrt{2}, \pi \dots$$

Real Numbers: all numbers that can be represented by a point on the number line; the set of rational numbers and irrational numbers.

Absolute Value: of a real number a , $|a|$, is the distance between a and zero on the number line.

$$\text{For any real number } a, |a| = a \text{ if } a \geq 0 \text{ and } |a| = -a \text{ if } a < 0$$

Progress Check 6 Evaluate each expression.

a. $|\!-\sqrt{2}|$

b. $-|-5|$

c. $|0|$

Commutative Property	Addition $a + b = b + a$	Multiplication $a \cdot b = b \cdot a$
Associative Property	$a + (b + c) = (a + b) + c$	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Identity Property	$a + 0 = 0 + a = a$	$a \cdot 1 = 1 \cdot a = a$
Inverse Property	$a + (-a) = (-a) + a = 0$	$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$
Distributive Property	$a(b + c) = ab + ac$	$(a + b)c = ac + bc$

Ordering Real Numbers

less than	less than or equal to	greater than	greater than or equal to
$<$	\leq	$>$	\geq

Adding Real Numbers

Like Signs: add the two numbers (absolute values) & keep the common sign.

Unlike Signs: subtract the two numbers (absolute values) & keep the sign of the larger (absolute value) number.

Subtracting Real Numbers

We define subtraction in terms of addition. If a and b are any real numbers, then

$$a - b = a + (-b)$$

Multiplication & Division of Real Numbers

Same Signs: multiply (or divide) the two numbers and the sign of the product (or quotient) is **positive**.

Different Signs: multiply (or divide) the two numbers and the sign of the product (or quotient) is **negative**.

Sign Rule

A product of nonzero factors is **positive** if the number of negative factors is **even**. The product is **negative** if the number of negative factors is **odd**.

Factor: is each number (or variable) in a product.

ex: $5(-2)$, 5 is a factor and -2 is a factor ex: $2x$, 2 is a factor and x is a factor

Powers: $a^n = \underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_{n \text{ factors}}$

n is the **exponent** or power and a is called the **base**.

Progress Check 10 Multiply (or evaluate).

a. $(-5)^3$

b. -2^6

c. $(-2)^6$

Multiplication and Division Involving the Number 0

The product of any number and zero is zero, so

$$a \cdot 0 = 0 \cdot a = 0$$

Division involving 0

1. 0 divided by any nonzero number is 0

$$\text{ex: } 0 \div (-2) = 0 \text{ or } \frac{0}{-2} = 0$$

2. Division by 0 is undefined

$$\text{ex: } (-2) \div 0 = \text{undefined or } \frac{-2}{0} = \text{undefined}$$

Reciprocals: when the product of two numbers is 1, then the numbers are called reciprocals of each other.

$$\text{ex: reciprocal of } 3 \rightarrow \frac{1}{3}$$

$$\text{ex: reciprocal of } \frac{2}{5} \rightarrow \frac{5}{2}$$

Definition of Division

If a and b are real numbers with $b \neq 0$, then

$$a \div b = a \cdot \frac{1}{b}$$

Order of Operations (P E MD AS)

1. Perform all operations within grouping symbols, such as parentheses, first. If there is more than one symbol of grouping, simplify the innermost symbol of grouping first, and simplify the numerator and denominator of a fraction separately.
2. Evaluate powers of a number.
3. Multiply or divide working from left to right.
4. Add or subtract working left to right.

Progress Check 13 Evaluate $8 - 5(6 - 10)^2$

Progress Check 14 Evaluate on a graphing calculator.

a. $\frac{3 - (-1)}{-5 - 2}$

b. $|4 - 3^2|$

Review ALL the EXAMPLES in this section!

THINK ABOUT IT 1.1

1. A common student error is to assume that $-a$ must represent a negative number. Explain why this assumption is incorrect.
3. The expression $-(-8) - 3$ illustrates three uses of the $-$ symbol. Describe the three uses.
4. Find the set of all replacements for a that make the statement true.
 - a. $|a| = 2$
 - b. $|a| = -3$
 - c. $|a| = a$
 - d. $|a| = -a$
 - e. $|a| = |-a|$

Chapter 1.2 - Algebraic Expressions and Geometric Formulas

Variable: is a symbol that may be replaced by different numbers in a particular problem.

ex: $y+7$, y is the variable or $9z$, z is the variable

Constant: is a symbol that represents the same number throughout a particular problem.

ex: $y+7$, 7 is the constant

Algebraic Expression: is an expression that combines variables and constants using the operations of arithmetic.

ex: $4x^2 - 3x + 8$ or $\frac{1}{2}gt^2$

Progress Check 1 Evaluate each expression, given that $x = -2$, $y = 3$, and $z = -4$.

a. $4x - 3y$

b. $-y^2 + 2xz^3$

Subscripts: are small numbers to the bottom-right of a variable which are used to denote various values of the same variable.

ex: $m = \frac{y_2 - y_1}{x_2 - x_1}$ (slope formula)

Progress Check 2 Using the slope formula above evaluate m when $x_1 = 3$, $y_1 = -4$, $x_2 = 7$, and $y_2 = -10$.

Terms: parts of an algebraic expression separated by addition or subtraction.

ex: $-x^2 + 5x + 6x^2 - 4x + 9$, there are five terms in this expression

Like Terms: terms that have identical variable factors.

ex: $\underline{-x^2}$ $\underline{+5x}$ $\underline{+6x^2}$ $\underline{-4x}$ $+9$

Coefficients: is the constant (number) multiplied with the variable in a term.

ex: $-x^2 + 5x + 6x^2 - 4x + 9$, -1 , 5 , 6 , -4 are the coefficients of the terms respectively

Progress Check 4 Simplify by combining like terms if possible.

c. $2p - 3p + 4p$

d. $-x^2 + 6x + 6x^2 - x$

Progress Check 5 Remove the symbols of grouping and combine like terms (or simplify).

a. $-(x+y) + 3(x-y)$

b. $5x - 4[x - (1+x)]$

Formula: is an equality statement that expresses the relationship between two or more variables.

Square

$$P = 4s$$

$$A = s^2$$

Rectangle

$$P = 2l + 2w$$

$$A = l \cdot w$$

Triangle

$$P = a + b + c$$

$$A = \frac{1}{2}bh$$

Circle

$$C = 2\pi r$$

$$A = \pi r^2$$

Rectangular Solid

$$V = l \cdot w \cdot h$$

Cylinder

$$V = \pi r^2 h$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

Progress Check 7 A circular backyard aboveground pool has a diameter of 20 ft.

- a. What volume of water (to the nearest cubic foot) will it take to fill the pool to a depth of 4.3 ft?

Progress Check 8 Translate each statement into an algebraic expression.

- a. \$5 more than the retail price p . b. 8 percent of the product of a and b .

Review the EXAMPLES in the text which are before each of the PROGRESS CHECK's in these notes (they will have the same number as the Progress Checks)!

THINK ABOUT IT 1.2

- Explain the difference in meaning between $-a^2$ and $(-a)^2$.
- a. Use the distributive property to show that $2x - 2x$ equals 0 for all values of x .
- If $xy = x$, what are the possible values for x and y ?
- In part a try the number trick with at least one number. Then show algebraically why the trick works.
 - Choose a number.
Add three.
Multiply by two.
Add six.
Divide by two.
Subtract your original number.
Your result is six.

OPERATIONS	VERBAL EXPRESSION	ALGEBRAIC EXPRESSION
Addition	The sum of a number and 1	$x+1$
	The sum of -2 and 1	$-2+1$
	A number plus 2	$y+2$
	-7 plus 2	$-7+2$
	4 more than a number	$z+4$
	4 more than 6	$6+4$
	Add 5 and a number	$5+n$
	Add 5 and -1	$5+(-1)$
Subtraction	The difference of a number and 6	$d-6$
	The difference of 8 and 6	$8-6$
	A number minus 5	$w-5$
	-9 minus 5	$-9-5$
	A number decreased by 4	$t-4$
	2 decreased by 4	$2-4$
	3 less than a number	$b-3$
	3 less than 12	$12-3$
	Subtract a number and 11	$x-11$
	Subtract -7 and 11	$-7-11$
Subtract a number from 11	$11-p$	
Subtract -3 from 11	$11-(-3)$	
Multiplication	The product of a number and 5	$5c$
	The product of -4 and 5	$(-4)(5)$ or $-4 \cdot 5$ or $-4(5)$
	4 times a number	$4n$
	4 times 9	$(4)(9)$
	A number multiplied by 7	$7y$
	-8 multiplied by 7	$(-8)(7)$
	Twice a number	$2x$
	Double a number	$2x$
	$\frac{1}{3}$ of a number	$\frac{1}{3}x$
$\frac{1}{3}$ of 9	$\frac{1}{3}(9)$	
Division	The quotient of a number and 8	$z \div 8$ or $\frac{z}{8}$
	The quotient of -16 and 8	$\frac{-16}{8}$
	A number divided by 2	$\frac{t}{2}$
	10 divided by 2	$\frac{10}{2}$

Chapter 1.3 - Integer Exponents

Laws of Integer Exponents

LAWS of INTEGER EXPONENTS	
Let a and b be any nonzero real numbers, and m and n be any integers.	
$a^0 = 1$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
$a^1 = a$	$\frac{a^m}{a^n} = a^{m-n}$
$a^m \cdot a^n = a^{m+n}$	$a^{-n} = \frac{1}{a^n}$
$(a^m)^n = a^{m \cdot n}$	$\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$
$(ab)^n = a^n b^n$	$\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

Progress Check 1 Simplify by laws of exponents.

a. $3^4 \cdot 3$

b. $(3^4)^2$

c. $(4 \cdot 2)^3$

d. $\frac{3^4}{3^2}$

e. $\frac{3^2}{3^4}$

Progress Check 2 Simplify by laws of exponents.

a. $(-4x)^2(9x^5)$

b. $\frac{14xy^5}{21x^4y^4}$

Progress Check 3 Evaluate each expression.

a. $-5x^0$

b. -5^0

c. 4.72×10^0

Progress Check 4 Evaluate each expression.

a. 7^{-1}

b. $(-3)^{-4}$

c. $\left(\frac{2}{5}\right)^{-3}$

Progress Check 6 Simplify and write the result using only positive exponents.

a. $2^{-5} \div 2^3$

b. $3y^{-5} \cdot y^4$

c. $\left(\frac{2x}{5}\right)^{-3}$

d. $(6y^{-4})^{-1}$

e. $\frac{3^2 x^{-4} y^{-5}}{3^{-1} x^4 y^{-3}}$

Progress Check 7 Simplify and write the result using only positive exponents.

a. $\frac{2}{2^{-3}}$ **This is Example 7b.** $\frac{3^{-2}x^2y^{-3}}{3x^{-5}y}$

Progress Check 8 Simplify the given expressions.

a. $a^x \div a^{-x}$ b. $3^n \cdot 3^n$ c. $\frac{x^{3n}}{x^{2n}}$

Try 1. $\frac{3y^{m+2}}{-9y^m}$ 2. $a^x \cdot a^{-x}$ 3. $\frac{x^n}{x^{2n}}$

Progress Check 9 Write each number in scientific notation.

a. 615 million b. -0.09

Progress Check 10 Write each number in scientific notation.

a. 9.2×10^9 b. -2.7×10^{-6}

Review the EXAMPLES in the text which are before each of the PROGRESS CHECK's in these notes (they will have the same number as the Progress Checks)!

Chapter 1.4 - Products of Algebraic Expressions

Objectives:

- ✓ Use the **distributive property** to multiply algebraic expressions.
- ✓ Use the **FOIL** method to multiply **two expressions** that each **contain two terms**.
- ✓ Use **special product** formulas to find certain products.

Progress Check 1 Find each product.

a. $-2x^2(3x^2 - x + 9)$ b. $(7y - x)3y$

Progress Check 2 Divide $-12n^2x^2 + 2nx^2$ by $4nx$.

Progress Check 3 Find each product.

a. $(3y - 4)(7y - 2)$ b. $(a^2 + 3)(a^2 - 5)$

Progress Check 4 Find each product.

a. $(4x^2 + 3x - 2)(x + 3)$

Progress Check 7 Multiply using the FOIL method

a. $(k - 8)^2$ b. $(x + h)^2$ c. $(3y + 5)(3y - 5)$

Special Products Formulas

1. $(a + b)(a - b) = a^2 - b^2$

2. $(a + b)^2 = a^2 + 2ab + b^2$

3. $(a - b)^2 = a^2 - 2ab + b^2$

Progress Check 8 Use a special product formula to find each product.

a. $(2x + 7)^2$ b. $(2x + 7)(2x - 7)$ c. $(3m - 5n)^2$

Progress Check 9 Simplify $\frac{3(x + h)^2 + 7 - (3x^2 + 7)}{h}$

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Chapter 1.5 – Linear Equations and Literal Equations**Objectives:**

- ✓ Solve linear equations.
- ✓ Find the value of a variable in a formula when given values for the other variables.
- ✓ Solve a given formula or literal equation for a specified variable.

Progress Check 2 Solve the equation.

$$2(5-3y) = 1-8y$$

Progress Check 3 Solve the equation.

$$\frac{2x-5}{6} = \frac{x-6}{9}$$

Progress Check 4 Solve the equation.

$$2(x+1) = 3(x+2) - x$$

Progress Check 5 Find the value of r in the formula $A = P(1+rt)$ if $A = \$6,240$, $P = \$4000$, and $t = 7$ years.

Progress Check 7 Solve $S = t - \frac{1}{2}gt^2$ for g .

Progress Check 8 Solve $m = \frac{y-b}{x}$ for y .

Progress Check 9 Solve $5x - 2y = 30$ for the indicated letter.

a. For x

b. For y

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Chapter 1.6 – Applications of Linear Equations

Objectives:

- ✓ Solve word problems by translating phrases and setting up and solving equations.
- ✓ Solve problems involving geometric figures, annual interest, uniform motion, liquid mixtures, and proportions.

To Solve a Word Problem

1. Read the problem several times.

- determine and write down what you are asked to find
- note what information is given
- if possible, display the given information in a sketch or chart

2. Let a variable represent an unknown quantity.

- write down precisely what the variable represents
- if there is more than one unknown, represent these unknowns in terms of the original variable

3. Set up an equation.

- express the relationship between the quantities in the problem

4. Solve the equation.

5. Answer the question.

6. Check the answer.

- interpret the solution in the context of the word problem

Progress Check 1 The total cost (including tax) of a new car is \$14,256. If the sales tax rate is 8 percent, how much is paid in taxes?

Progress Check 2 The sum of two consecutive integers is 147. Find the integers.

Geometric Problems

- For geometric figure problems use perimeter, area, and volume formulas from section 1.2
- For triangle problems remember the sum of the angle measures in a triangle is 180° and a **right triangle** contains a 90° angle
- For angle problems remember that any two angles whose measures add up to 90° are called **complementary** and any two angles whose measures add up to 180° are called **supplementary**.

Progress Check 3 In a right triangle the measure of one of the acute angles is 36° greater than the other. What is the measures if the larger acute angle?

Annual Interest Problems

Use $I = P \cdot r$ where P represents principal (amount invested), r represents the annual interest rate (a percentage which must be changed to decimal form), and I represents the amount of interest earned in one year.

Progress Check 4 How should a \$200,000 investment be split so that the total annual earnings are \$18,000, if one portion is invested at 10 percent annual interest and the rest at 6 percent?

Uniform Motion Problems

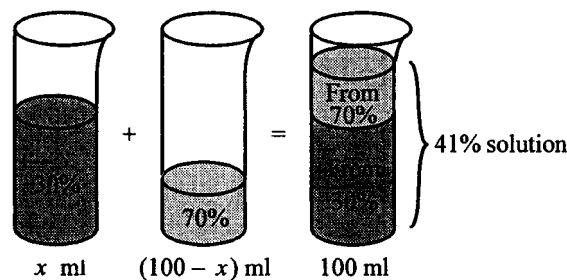
Use the formula $d = r \cdot t$, where d represents the distance traveled in time t by an object moving at a constant rate r . This formula applies to objects moving at a constant (or uniform) speed and to objects whose average speed is involved. A chart and a sketch is recommended for this type of problem.

Progress Check 5 On a video display, an air traffic controller notices two planes 120 miles apart and flying toward each other on a collision course. One plane is flying at 500 mi/hour; the other is flying at 300 mi/hour. How much time is there for the controller to prevent a crash?

Liquid Mixture Problems

Use (percent of an ingredient) \cdot (amount of solution) = (amount of the ingredient) **and**

(amount of the ingredient in 1st solution) + (amount of the ingredient in 2nd solution) = (amount of the ingredient in new solution)



Progress Check 6 One metal contains 30 percent gold by weight and the rest silver. Another contains 50 percent gold by weight and the rest silver. They will be melted down and mixed together to form a new alloy that is 35 percent gold. How much of each should be used to form 5 lb of the new alloy?

Proportion Problems

A **ratio** is a comparison of two quantities by division, and a **proportion** is a statement that two ratios are equal.

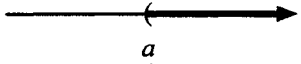
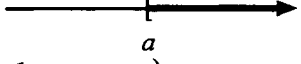
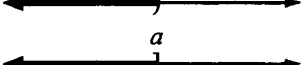
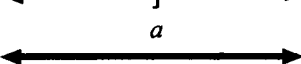
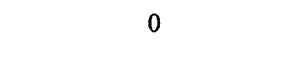
Progress Check 7 If an idling car uses 35 oz. of gasoline in 50 minutes, how long to the nearest minute must it idle to use 1 gal of gas? One gallon is 128 oz.

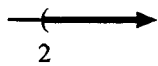
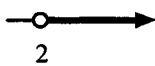
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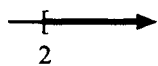
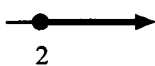
Chapter 1.7 – Linear Inequalities in One Variable

Objectives:

- ✓ Specify solution sets of linear inequalities by using graphs and interval notation.
- ✓ Solve linear inequalities by applying properties of inequalities.

Set Notation	Graph	Interval Notation
$\{x: x > a\}$		(a, ∞)
$\{x: x \geq a\}$		$[a, \infty)$
$\{x: x < a\}$		$(-\infty, a)$
$\{x: x \leq a\}$		$(-\infty, a]$
$\{x: x \text{ is a real number}\}$		$(-\infty, \infty)$

 is equivalent to 

 is equivalent to 

$a < b$ is equivalent to $b > a$

$2 < 7$ is equivalent to $7 > 2$

Progress Check 2 Solve $-3x - 7 > 5$. Express the solution set graphically, in set notation, and in interval notation.

Progress Check 3 Solve $5(x - 1) > 2x + 1$. Express the solution set graphically, in set notation, and in interval notation.

Progress Check 4 Solve $6 - 2x \geq x$. Express the solution set graphically, in set notation, and in interval notation.

Progress Check 5 Solve $x < x - 3$.

Review the **EXAMPLES** in the text which are before each of the **PROGRESS CHECK**'s in these notes (they will have the same number as the Progress Checks)!

Chapter 1.8 – Compound Inequalities

Objectives:

- ✓ Solve compound inequalities involving *and* statements.
- ✓ Solve compound inequalities involving *or* statements.

Type of Interval	Set Notation	Graph	Interval Notation
Open interval	$\{x: a < x < b\}$		(a, b)
Closed interval	$\{x: a \leq x \leq b\}$		$[a, b]$
Half-open interval	$\{x: a \leq x < b\}$		$[a, b)$
	$\{x: a < x \leq b\}$		$(a, b]$

Solution of *And* Inequalities

To solve a compound inequality involving *and*:

1. Solve separately each inequality in the compound inequality.
2. Find the intersection (*only what belongs to both, or the overlap*) of the solution sets of the separate inequalities.

Progress Check 3 Solve $x - 4 < 0$ and $2x \geq 3x - 12$. Express the solution set graphically and in interval notation.

Solution of *Compact Form* Inequalities

To solve a compound inequality in *compact form* the goal is to isolate the variable in the middle member of the inequality.

Progress Check 4 Solve $-128 \leq 160 - 32t \leq 128$. Express the solution set graphically and in interval notation.

Progress Check 5 In a philosophy class an average from 74.5 up to but not including 79.5 results in a grade of C+. If your first three grades are 68, 83, and 75, find all possible grades on the fourth exam that would result in a grade of C+.

Solution of *Or* Inequalities

To solve a compound inequality involving *or*:

1. Solve separately each inequality in the compound inequality.
2. Find the union (*everything that belongs to either or both*) of the solution sets of the separate inequalities.

Progress Check 6 Solve $2x - 1 < -3$ or $2x - 1 > 3$. Express the solution set graphically and in interval notation.

Review the EXAMPLES in the text which are before each of the PROGRESS CHECK's in these notes (they will have the same number as the Progress Checks)!