Section 2.1 - *Input and Output*

Finding Output Values: Evaluating a Function
Evaluating a function means calculating the value of a function's output from a particular value of the input.

Evaluating a Function Using a Formula
If we have a formula for a function, we evaluate it by substituting the input value into the formula.

Finding Input Values: Solving Equations
Sometimes we know the output and we want to find the corresponding input value. If the function is given by a formula, the input values are solutions to an equation.

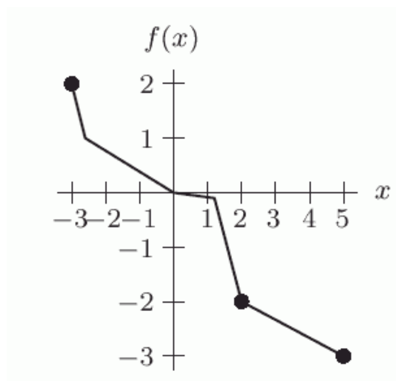
Finding Output and Input Values from Tables and Graphs

Chapter 2.1 – *Example on Output and Input Values from a Graph* *(NOT in Textbook)*
Use the following figure to evaluate:

a. $f(-3)$

b. $f(2)$

c. $f(x) = -3$



Chapter 2.1 – *Example 1 on pg. 62 in Text*
Remembering ***Section 1.1 - Example 2 on pg. 4 in Text*** - The number of gallons of paint needed to paint a house depends on the size of the house. A gallon of paint typically covers 250 square feet. So, the number of gallons of paint, *n*, is a function of the area to be painted, *A* sq.ft. and the formula is $n = f(A) = \frac{A}{250}$.

a. Evaluate the expression $f(20{,}000)$

Chapter 2.1 – *Example 3 on pg. 62 in Text*
Let $g(x) = \frac{x^2+1}{5+x}$. Evaluate the following expressions.

a. $g(3)$

b. $g(-1)$

c. $g(a)$

Chapter 2.1 – *Example 4 on pg. 63 in Text*

Let $h(x) = x^2 - 3x + 5$. Evaluate and simplify the following expressions.

a. $h(2)$

b. $h(a-2)$

c. $h(a)-2$

d. $h(a)-h(2)$

Chapter 2.1 – *Example 5 on pg. 63 in Text*

Use the cricket function $T = \frac{1}{4}R + 40$, introduced on ***page 3 in section1.1***, to find the rate *R*, at which the snowy tree cricket chirps when the temperature, *T*, is 76°F.

Chapter 2.1 – *Example 6 on pg. 63 in Text*

Suppose $f(x) = \frac{1}{\sqrt{x-4}}$.

a. Find an *x*-value that results in $f(x) = 2$.

b. Is there an *x*-value that results in $f(x) = -2$?

Chapter 2.1 – *Example 8 on pg. 64 in Text*

The table below shows the revenue, $R = f(t)$, received or expected, by the National Football League, NFL, from network TV as a function of the year, *t*, since 1975.

Year, *t* (since 1975)	0	5	10	15	20	25	30
Revenue, *R* (millions $)	201	364	651	1075	1159	2200	2200

a. Evaluate and interpret $f(25)$

b. Solve and interpret $f(t) = 1159$

Review ***Example 9 on pg. 65.***

Section 2.2 - *Domain and Range*

A function is often defined only for certain values of the independent variable. Also, the dependent variable often takes on only certain values. This leads to the following definitions:

> If $Q = f(t)$ then:
>
> - The **domain** of f is the set of input values, *t*, which yield an output value.
> - The **range** of f is the corresponding set of output values, *Q*.

Thus, the domain of a function is the set of input values, and the range is the set of output values. If the domain of a function is not specified, we usually assume that it is as large as possible—that is, all numbers make sense as inputs for the function. If a function is being used to model a real-world situation, the domain and range of the function are often determined by the constraints of the situation being modeled.

Choosing Realistic Domains and Ranges

When a function is used to model a real situation, it may be necessary to modify the domain and range.

Using a Graph to find the Domain and Range of a Function

A good way to estimate the domain and range of a function is to examine its graph. The domain is the set of input values on the horizontal axis which give rise to a point on the graph; the range is the corresponding set of output values on the vertical axis.

Using a Formula to find the Domain and Range of a Function

When a function is defined by a formula, its domain and range can often be determined by examining the formula algebraically.

Chapter 2.2 – ***Example 1 on pg. 69 in Text.***

What is the domain and range for the house painting function $n = f(A)$?

Chapter 2.2 – ***Example 2 on pg. 69 in Text***

Considering the formula $T = \frac{1}{4}R + 40$ from the cricket problem, what is the domain and range if:

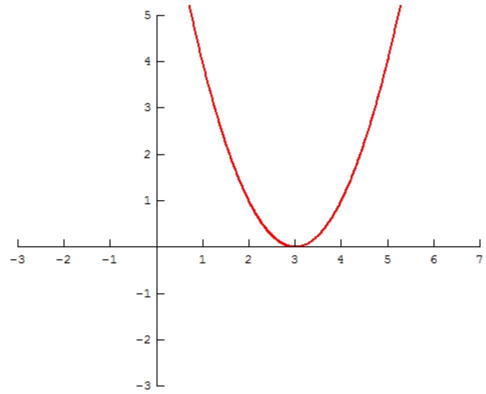
a. we are just looking at the formula algebraically?

b. we are using the formula to represent the temperature, *T*, as a function of a cricket's chirp rate, *R*?

Chapter 2.2 – *Examples on Domain and Range from a Graph of a Function* *(NOT in Textbook)*

1. What is the domain of this function?

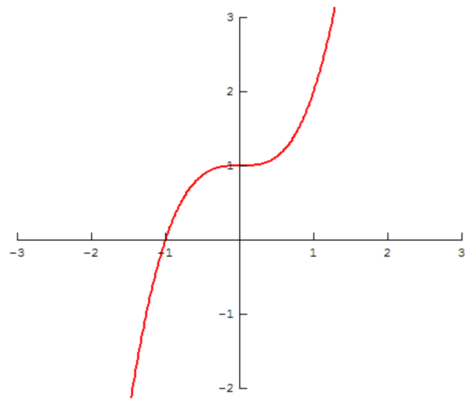
   What is the range of this function?



2. What is the domain of this function?

   What is the range of this function?

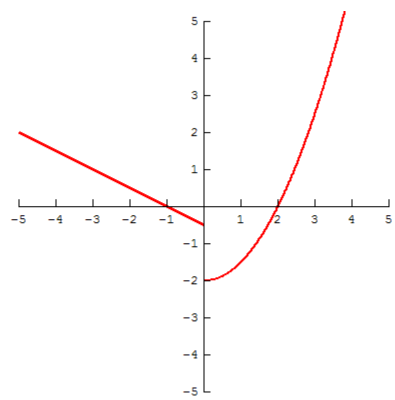
   What are the intercepts of this function?



3. What is the domain of this function?

   What is the range of this function?

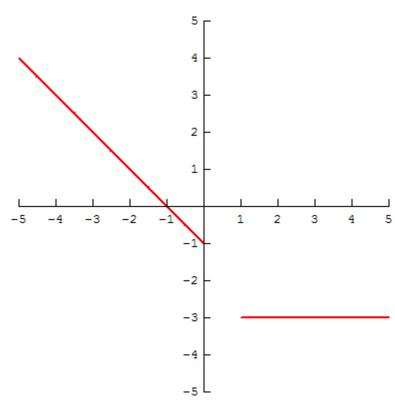
   What are the intercepts of this function?



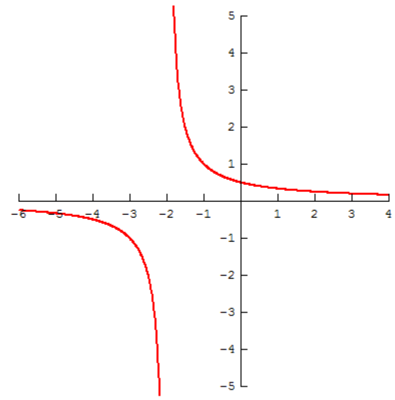
4. What is the domain of this function?

   What is the range of this function?

   What are the intercepts of this function?

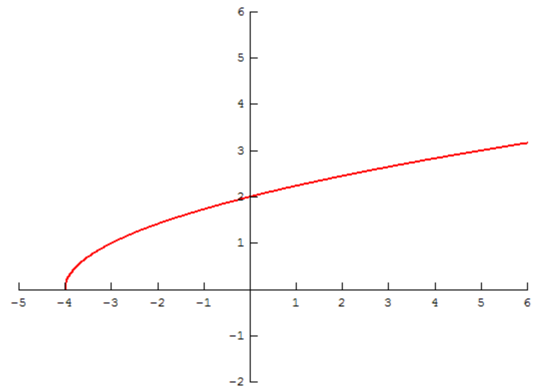


5. What is the domain of this function?



What is the range of this function?

6. What is the domain of this function?



What is the range of this function?

What are the intercepts of this function?

Chapter 2.2 – *Example 4 on pg. 71 in Text*

State the domain and range of *g*, where $g(x) = \frac{1}{x}$

Chapter 2.2 – *Example 5 on pg. 71 in Text*

Find the domain and range of the function $f(x) = \frac{1}{\sqrt{x-4}}$.

Section 2.3 - *Piecewise Defined Functions*

A function may employ different formulas on different parts of its domain. Such a function is said to be *piecewise defined*.

Chapter 2.3 – *Example 1 on pg. 73 in Text*

Graph the function $g(x)=\begin{cases} x+1 & \text{for } x \le 2 \\ 1 & \text{for } x > 2 \end{cases}$

Chapter 2.3 – *Example 2 on pg. 74 in Text*

A long-distance plan charges 99 cents for any call up to 20 minutes in length and 7 cents for each additional minute or part of a minute.

a. Use bracket notation to write a formula for the cost, *C*, of a call as a function of its length t in minutes.

b. Graph the function.

c. State the domain and range of the function.

The Absolute Value Function

The absolute value function is defined by $f(x)=|x|=\begin{cases} x & \text{for } x \ge 0 \\ -x & \text{for } x < 0 \end{cases}$

Example for the absolute value function

a. Make a table for the absolute value function.

b. Graph the function.

Section 2.4 - *Composite and Inverse Functions*

Composition of Functions
Two functions may be connected by the fact that the output of one is the input of the other.

Inverse Functions
The roles of a function's input and output can sometimes be reversed. Suppose we have the function $y = f(x)$, where *x* is the input and *y* is the output. If we define a new function, $x = g(y)$, this tells us the value of *x* given the value of *y* instead of the other way round. The functions f and *g* are called *inverses* of each other. A function which has in inverse is said to be *invertible*.

Inverse Function Notation
Let's say we have *y* is a function of *x*, then $y = f(x)$. To express that *x* is also determined by *y*, so that *x* is a function of *y*, we write $x = f^{-1}(y)$. The symbol f^{-1} is used to represent the function that gives the output *x* for a given input *y*.

Finding a Formula for the Inverse Function

Domain and Range of an Inverse Function
The input values of the inverse function f^{-1} are the output values of the function f. Thus, the domain of f^{-1} is the range of f.

The functions f and f^{-1} are called inverses because they "undo" each other when composed.

Chapter 2.4 – ***Example 1 on pg. 79 in Text***
Again, remembering ***Section 1.1 - Example 2 on pg. 4 in Text*** :

The number of gallons of paint needed to paint a house depends on the size of the house. A gallon of paint typically covers 250 square feet. So, the number of gallons of paint, *n*, is a function of the area to be painted, *A* square feet and the formula is $n = f(A) = \dfrac{A}{250}$.

If we wanted to find the cost, *C*, in dollars, to paint a room of area *A* square feet, we need to know the number, *n*, of gallons of paint required.

Now, if paint is \$30.50 a gallon, we have a function $C = g(n) = 30.50n$.

The **output** of $n = f(A) = \dfrac{A}{250}$ which is *n*, is the **input** of $C = g(n) = 30.50n$.

Find a formula for the cost, *C*, as a function of area, *A*.

C is a "function of a function", or a **composite function**. If the function giving C in terms of A is called h, so $C = h(A)$, then we write $C = h(A) = g(f(A))$. The function h is said to be the *composition* of the functions f and g. We say f is the *inside* function and g is the *outside* function.

Chapter 2.4 – *Example 3 on pg. 80 in Text*
If $f(x) = x^2$ and $g(x) = 2x + 1$, find

a. $f(g(x))$

b. $g(f(x))$

Chapter 2.4 – *Example 4 on pg. 80 in Text*
Using $P = f(t)$, where P represents the population, in thousands, of birds on an island and t is the number of years since 2007:

a. What does $f(4)$ represent?

b. What does $f^{-1}(4)$ represent?

Chapter 2.4 – *Example 5 on pg. 81 in Text*
The cricket function, which gives the temperature, T, in terms of chirp rate, R, is
$T = f(R) = \frac{1}{4}R + 40$.
Find a formula for the inverse function, $R = f^{-1}(T)$.

Chapter 2.4 – *Example 6 on pg. 81 in Text*
Calculate the composite functions $f^{-1}(f(R))$ and $f(f^{-1}(T))$.

Chapter 2.4 – *Example on Inverse Functions from a Table and Finding a Formula for the Inverse Function* *(NOT in Textbook)*

The town of Jonesville has a population of 15,000 people in 1980 and grows by 200 citizens every year. Lets look at the population, *P*, as a function of time, *t* (in years since 1980), in terms of a table.

t	0	1	2	5	10
$P = f(t)$	15,000	15,200	15,400	16,000	17,000

a. Find a formula for the population of Jonesville.

b. When will the population of Jonesville reach 30,000?

c. What is the notation for the inverse function and find the formula for the inverse function.

d. Find and interpret $f(7)$

e. Find and interpret $f^{-1}(15{,}800)$

Chapter 2.4 – *Example on Output and Input Values from a Graph of an Invertible Function f* *(NOT in Textbook)*

Use the following figure to evaluate:

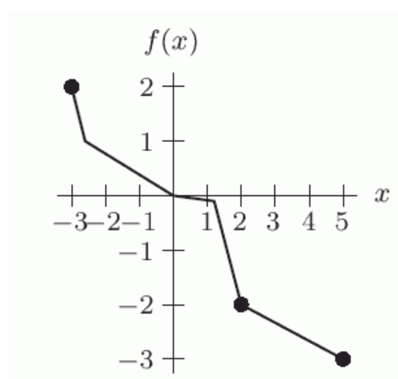
a. $f^{-1}(-3) = ?$

b. $f^{-1}(-2) = ?$

c. $f^{-1}(2) = ?$

d. $f^{-1}(?) = -3$

e. $f^{-1}(?) = 2$



Section 2.5 - *Concavity*

Concavity and Rates of Change

The graph of a linear function is a straight line because the average rate of change is a constant. However, not all graphs are straight lines; they may bend up or down. If the rate of change is increasing, the slope if the graph increases as the independent variable increases, so the graph bends upward. We say such graphs are *concave up*. Graphs can bend downward; we call such graphs *concave down*.

Summary: Increasing and Decreasing Functions; Concavity

- If f is a function whose rate of change increases, then the graph of f is **concave up**. That is, the graph bends upward.
- If f is a function whose rate of change decreases, then the graph of f is **concave down**. That is, the graph bends downward.

Chapter 2.5 – *Example on Output and Input Values from a Graph of an Invertible Function f*
(NOT in Textbook)

Let's consider the distances, in miles, traveled by two cyclists, Mike and Colby, as function of time, in hours.

a. Calculate the rate of change over each consecutive interval for each cyclist.
b. Graph the distance function for each cyclist.
c. State the concavity of each graph.
d. State the speed of each cyclist as increasing, decreasing, or constant.

***t*, time (hours)**	***d*, Mike's distance (miles)**	**Average speed $\Delta d/\Delta t$ (mph)**
0	0	
1	16	
2	35	
3	58	
4	85	

***t*, time (hours)**	***d*, Colby's distance (miles)**	**Average speed $\Delta d/\Delta t$ (mph)**
0	0	
1	30	
2	52	
3	72	
4	91	

Review ***Example 1 and Example 2 on pgs. 84 – 85***, and ***Figures 2.22 – 2.25 on pg. 86 (top)***.

Section 2.6 - *Quadratic Functions*

The **General Form** of a quadratic function is $y = f(x) = ax^2 + bx + c$.

Finding the Zeros of a Quadratic Function
Input values which make the output of a function equal to 0 are called *zeros* of *f*. It is easy to find the zeros of a quadratic function if its formula can be factored.

The **Factored Form** of a quadratic equation is $f(x) = a(x-r)(x-s)$, zeros at $x = r$ and $x = s$.

We can also find the zeros of a quadratic function by using the quadratic formula.
The **Quadratic Formula** is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The *zeros* of a function occur at the *x*-intercepts of its graph. Not every function has *x*-intercepts; therefore the function would have no real zeros.

Concavity and Quadratic Functions
Unlike a linear function, whose graph is a straight line, a quadratic function has a graph which is either concave up or concave down.

Chapter 2.6 – *Example on pg. 88 in Text*
A baseball is "popped" straight up by a batter. The height of the ball above ground is given by the function $y = f(x) = -16t^2 + 64t + 3$, where *t* is time in seconds after the ball leaves the bat and *y* is in feet. Although the path of the ball is straight up and down, the graph of its height as a function of time is concave down. The ball goes up fast at first and then more slowly because of gravity.

a. Graph the function. (*window:* $-1 \le x \le 5$ & $-10 \le y \le 80$)

b. What is the balls initial height?

c. When does the ball hit the ground?

d. What is the maximum height the ball reaches?

e. Domain and range?

Chapter 2.6 – *Example 1 on pg. 89 in Text*
Find the zeros of the function $f(x) = x^2 - x - 6$.

Chapter 2.6 – *Example 2 on pg. 89 in Text*
Find the zeros of the function $f(x)=x^2-x-6$ using the quadratic formula.

Chapter 2.6 – *Example 3 on pg. 89 in Text*
Find the zeros of the function $f(x)=-\frac{1}{2}x^2-2$ using algebra. Graph this function. Explain what this tells you. What would happen if you used the quadratic formula?

Chapter 2.6 – *Example on a Quadratic Function that has One Zero* *(NOT in Textbook)*
Find the zeros of the function $f(x)=x^2-16$

Chapter 2.6 – *Example on a Quadratic Function that is Not Factorable* *(NOT in Textbook)*
Find the zeros of the function $f(x)=4x^2-4x-8$

Chapter 2.6 – *Example on Finding a Quadratic Function Given the Zeros* *(NOT in Textbook)*
Find two quadratic functions with zeros $x=1$ and $x=-5$.

Chapter 2.6 – *Example 4 on pg. 90 in Text*
Let $f(x)=x^2$. Find the average rate of change of f over the intervals of length 2 between $x=-4$ and $x=4$. What does this tell you about the concavity of the graph of f ?

Review ***Example 5 on pg. 91***