Section 5.2 - *Random Variables*

A **random variable** is a variable that takes on different numerical values which are determined by chance.

Example 5.1 pg. 233 – For each random experiment, define a random variable and identify the possible values of the variable.

a. It is assumed that the largest number of children is 6, count the number of children within a family.

b. Select a radial tire from the production line and determine the life of the tire, assuming no tire has ever lasted less than 20,000 miles or more than 85,000 miles.

c. Count the number of heads in two tossed of a fair coin.

A **discrete random variable** is a random variable that can take on a finite or countable number of values.

A **continuous random variable** is continuous if the value of the random variable can assume any value or an uncountable number of values between any two possible values of the variable.

Example 5.2 pg. 234 – For each random variable, state whether it is discrete or continuous.

a. The life of a 32 inch color picture tube.

b. The number of phone calls arriving at a college switchboard during the day.

c. The number of employees that are absent from work at a steel manufacturing plant during a summer day.

d. The thickness of a multivitamin.

5.3 Probability Distribution of a Discrete Random Variable

A **probability distribution** is a distribution which displays the probabilities associated with all the possible values of a random variable.

Characteristics of a Probability Distribution of a Discrete Random Variable

1. The probability associated with a particular value of a discrete random variable of a probability distribution is always a number between 0 and 1 inclusive.
2. The sum of all the probabilities of a probability distribution must always be equal to one.

Example 5.3 on pg. 237 in the Text - Consider the experiment of tossing a fair coin until a head appears or until the coin has been tossed three times, whichever comes first.

a. Construct a probability distribution for the ***number of tails***.
b. Construct a probability histogram for this probability distribution.
c. Using the probability histogram, what is the probability of getting no tails?

Here is the experiment: toss a fair coin until a head appears or until the coin has been tossed 3 times.

First list all possible <u>outcomes</u> - there are four possible outcomes:

H	Head on the 1st toss **– Experiment over!**
T H	Tail on the 1st toss and Head on the 2nd toss **– Experiment over!**
T T H	Tail on the 1st two tosses and a Head on the 3rd toss **– Experiment over!**
	or
T T T	Tail on all three tosses **– Experiment over!**

Now what do we want to consider?
The number of tails which are: 0 tails, 1 tail, 2 tails, or 3 tails

Head on the 1st toss (H) = **0 tails**
Tail on the 1st toss and Head on the 2nd toss (TH) = **1 tail**
Tail on the 1st two tosses and a Head on the 3rd toss (TTH) **OR** = **2 tails**
Tail on all three tosses (TTT) = **3 tails**

Probability of getting a head is ½ and of getting a tail is ½

Letting *X* = the number of tails, we can construct the following table:

Number of Tails X	Possible Outcomes	Probability P(X)
0	H	$\frac{1}{2}$
1	**T** H	$\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$
2	**T T** H	$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{8}$
3	**T T T**	$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{8}$

Notice that the probabilities add up to 1: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}=1$

Now let's construct the probability distribution for part a.

a. **Probability Distribution of Number of Tails**

Number of Tails X	**Probability P(X)**
0	$\frac{1}{2}$
1	$\frac{1}{4}$
2	$\frac{1}{8}$
3	$\frac{1}{8}$

b. The probability histogram is constructed by scaling the value of X along the horizontal axis and P(X) along the vertical axis.

Probability Histogram of Number of Tails

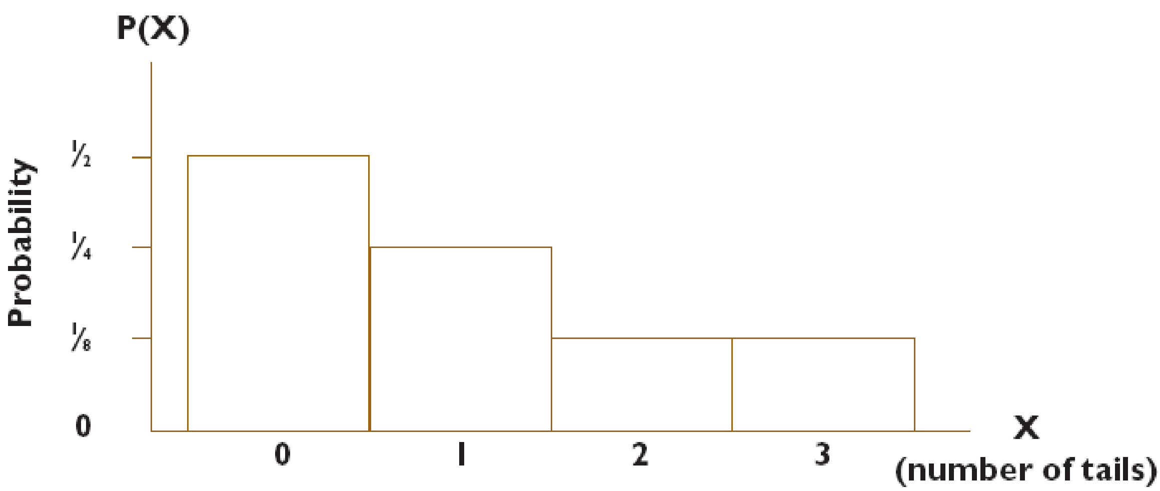

FIGURE 5.3

c. From the histogram, the probability of getting no tails is equal to the height of the bar graph pertaining to 0 tails which is ½ or 0.5. So the probability is ½ or 0.5.

Review Example 5.4 on pg. 239 in the Text

5.5 Binomial Probability Distribution

A **binomial experiment** satisfies the following four conditions:

1. There are *n identical trials*.

- A binomial distribution is the result of a probability experiment that has been repeated a predetermined number of *n* times, and each repetition (trial) of the experiment is identical
    - Such as tossing a coin twenty times

2. The *n* identical trials are *independent*.

- Each outcome (trial) is independent and mutually exclusive
    - For example: In the experiment of tossing a coin *n* times, the outcome of each toss (trial) is independent of any other toss

3. The outcome for each trial can be classified as either a *success* or a *failure*.

- Each outcome is classified in one of **two possibilities**, "success or "fail"
    - The determination as to whether an outcome is a success or failure is a function on how the question is asked
    - Success generally means a positive response to the question
- See top of pg. 253
    - the toss of a coin is either a *head* or a *tail*
    - the selection of a possible answer for a question on a multiple choice test is either *correct* or *incorrect*
    - the toss of a die results in an outcome which is either *a 5* or *not a 5*
    - a new drug will either be *effective* or *not effective*

4. The *probability of a success* is the *same* for *each trial*.

- The probability of success is the same for each trial
    - Meaning, in a coin tossing experiment, the probability of landing on Heads is the same for each toss (trial) of the coin

Binomial Probability Formula

For a binomial experiment, the probability of getting **s** successes in **n** trials is computed using the binomial probability formula. This formula is written as:

$$P(\textbf{s} \text{ successes in } \textbf{n} \text{ trials}) = {}_nC_s \cdot p^s \cdot q^{(n-s)}$$

where:

n= number of independent trials

s = number of successes

(n – s) = number of failures

${}_nC_s$ = the number of ways "s" successes can occur in "n" trials

p = the probability of a success for one trial

q = the probability of a failure for one trial = 1 – p

Or you can use the built-in functions of your TI83/84 calculator: $binompdf\,(n,\ p,\ s)$

[2nd] [DISTR] [0: binompdf (] [***n***] [**,**] [***p***] [**,**] [***s***]) [ENTER]

Example 5.9 on pg. 245 – Consider the experiment of tossing a fair coin five times, where we are interested in getting a head. Can this experiment be classified as a binomial experiment?
(Does it satisfy the four conditions?)

Review Examples 5.10 and 5.11 on pgs. 254-256

Example 5.14 pg. 256 - Use the binomial probability formula
P (**s** successes in **n** trials) = ${}_nC_s \cdot p^s \cdot q^{(n-s)}$ and the calculator, to determine the probability of getting 3 successes in 4 trials if p = $2/3$

Example *(NOT in text)* **–** Answer the following if we consider numbers from 0 to 10?

a. More than 7

b. Less than 3

c. At most 4

d. At least 6

Example *(NOT in text)* – A fair coin is tossed 10 times, where a success is getting a tail on a single toss of the coin. Calculate the probability of:

a. Getting three tails (s = 3)
b. Getting at most 1 tail

a. n= number of independent trials = 10
s = number of successes = number of tails = 3
(n – s) = number of failures = 10 – 3 = 7
${}_nC_s$ = the number of ways "s" successes can occur in "n" trials = ${}_{10}C_3$ = 120
p = the probability of a success for one trial = ½
q = the probability of a failure for one trial = 1 – p = 1 – ½ = ½

P (**s** successes in **n** trials) = ${}_nC_s \cdot p^s \cdot q^{(n-s)}$

P(getting 3 tails) $= {}_{10}C_3 \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^7 = 120\left(\frac{1}{8}\right)\cdot\left(\frac{1}{128}\right) = \frac{120}{1024}$
= **0.1172**

OR $= binompdf\left(10,\ \tfrac{1}{2},\ 3\right)$
= **0.1172**

b. Getting at most 1 tail - means getting 1 tail or less or 0 tail and 1 tail

P(at most 1 tail) = P(0 tail) + P(1 tail)
s = 0 s = 1

$$= {}_{10}C_0 \cdot \left(\frac{1}{2}\right)^0 \cdot \left(\frac{1}{2}\right)^{10} + {}_{10}C_1 \cdot \left(\frac{1}{2}\right)^1 \cdot \left(\frac{1}{2}\right)^9$$

= 0.0010 + 0.0098
= **0.0108**

OR $= binompdf\left(10,\ \frac{1}{2},\ 0\right) + binompdf\left(10,\ \frac{1}{2},\ 1\right)$ = **0.0108**

Example 5.15 pg. 257 - Each year the FBI reports the probability of a car being stolen. In a recent report, the FBI states that the probability a new car will be stolen during the year is 1 out of 75. If you and your three friends own new cars, what is the probability that none of these cars will be stolen this year?

n= number of independent trials = total number of cars =
s = number of successes = number of stolen cars =
(n – s) = number of failures =
${}_nC_s$ = the number of ways "s" successes can occur in "n" trials =
p = the probability of a success for one trial = probability of a car being stolen =
q = the probability of a failure for one trial = 1 – p =
P (**s** successes in **n** trials) = ${}_nC_s \cdot p^s \cdot q^{(n-s)}$

Example 5.16 pg.257 - A student is going to guess at the answers to all questions on a five question multiple choice test where there are four choices for each question. Calculate the probability of:

a. Guessing three correct answers
b. Guessing five correct answers
c. Guessing at most two correct answers
d. Guessing at least four correct answers

n= number of independent trials = number of questions on the test =
s = number of successes = number of correct answers =
(n – s) = number of failures =
${}_nC_s$ = the number of ways "s" successes can occur in "n" trials =
p = the probability of a success for one trial = probability of guessing a correct answer =
q = the probability of a failure for one trial = 1 – p = 1 – ¼ =
P (**s** successes in **n** trials) = ${}_nC_s \cdot p^s \cdot q^{(n-s)}$

a. Guessing three correct answers

b. Guessing five correct answers

c. Guessing at most two correct answers

d. Guessing at least four correct answers

Review Example 5.17 pg. 260-261