Section 1.1 - *Functions and Function Notation*

A function is a relationship between two quantities. If the value of the first quantity determines exactly one value of the second quantity, we say the second quantity is a function of the first.

A **function** is a rule which takes certain numbers as inputs and assigns to each input number exactly one output number. The output is a function of the input.

The inputs and outputs are also called *variables***.**

A function can be described using words, data in a table, points on a graph, or by a formula.

When we use a function to describe an actual situation, the function is referred to as a **mathematical model**. Such models can be powerful tools for understanding phenomena and making predictions.

Function Notation:

To indicate the quantity Q is function of t, we write:

Q is a function of t

Q equals "f of t"

$Q = f(t)$

Thus applying the **rule** f to the **input** value t, gives **output** value, $f(t)$
So, **output** = $f(t)$ and/or **output** = Q and **input** = t
Q is the **dependent variable** and t is the **independent variable**.

$$\text{Output} = f(\text{Input})$$

$$\text{Dependent} = f(\text{Independent})$$

- Functions don't have to be defined by formulas.
- It is possible for two quantities to be related and yet for neither quantity to be a function of the other.

How to Tell if a Graph Represents a Function: Vertical Line Test

Vertical Line Test: If there is a vertical line which intersects a graph in more than one point, then the graph does not represent a function.

Chapter 1.1 – *Example 1 on pg. 2 in Text*

It is a surprising biological fact that most crickets chirp at a rate that increases as the temperature increases. For the snowy tree cricket, the relationship between temperature and chirp rate is so reliable that this type of cricket is called the thermometer cricket. We can estimate the temperature (in degrees Fahrenheit) by counting the number of times a snowy tree cricket chirps in 15 seconds and adding 40.

The rule used to find the temperature ***T*** (in ° F) from the chirp rate ***R*** (in chirps per minute) is an example of a function.

The **input** is ________________ and **output** is ________________.

Describe the function using:

1. WORDS:

2. TABLE:

3. GRAPH:

4. FORMULA:

Chapter 1.1 – *Example 2 on pg. 4 in Text*

The number of gallons of paint a house depends on the size of the house. A gallon of paint typically covers 250 square feet. Thus, the number of gallons of paint, *n*, is a function of the area to be painted, *A* sq. ft. We write $n = f(A)$.

a. Find a formula for *f*

b. Explain in words what the statement $f(10{,}000) = 40$ tells us about painting houses.

Chapter 1.1 – *Example 4 on pg. 5 in Text*

The average monthly rainfall, *R*, a Chicago's O'Hare airport is given in Table 1.2, where time, *t*, is in months and $t = 1$ is January, $t = 2$ is February, and so on. The rainfall is a function of the month, so we write $R = f(t)$. However there is no equation that gives *R* when *t* is known. Evaluate $f(1)$ and $f(11)$. Explain what your answers mean.

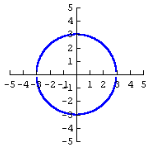
Average monthly rainfall at Chicago's O'Hare airport

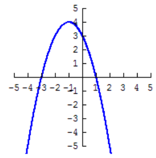
Month, *t*	1	2	3	4	5	6	7	8	9	10	11	12
Rainfall, *R* (inches)	1.8	1.8	2.7	3.1	3.5	3.7	3.5	3.4	3.2	2.5	2.4	2.1

*The example states that the rainfall is a function of the month. Why do we know this from looking at the table? ***Is the month a function of the rainfall*** (or is *t* a function of *R* or $t = f(R)$)? ***Explain***.

Chapter 1.1 – *Example* (*similar to Example 6 on pg. 6 in text*)

In which of the graphs below could *y* be a function of *x*?





Review ***Example 5 on pg. 5*** and ***Example 6 on pg. 6 in Text.***

Section 1.2 - *Rate of Change*

Rate of Change of a Function

The **average rate of change** or **rate of change**, of Q with respect to t over an interval is:

Average rate of change over an interval = $\dfrac{Change\ in\ Q}{Change\ in\ t} = \dfrac{\Delta Q}{\Delta t}$

The average rate of change of the function $Q = f(t)$ over an interval tells us how much Q changes, on average, for each unit change in t within that interval. On some parts of the interval, Q may be changing rapidly, while on other parts Q may be changing slowly. The average rate of change evens out these variations.

Increasing and Decreasing Functions

If $Q = f(t)$ for t in the interval $a \le t \le b$,

- f is an **increasing function** if the values of f increase as t increases on this interval.
- f is a **decreasing function** if the values of f decrease as t increases on this interval.

If $Q = f(t)$,

- If f is an increasing function, then the average rate of change of Q with respect to t is positive on every interval.
- If f is a decreasing function, then the average rate of change of Q with respect to t is negative on every interval.

In general, we can identify an increasing or decreasing function from its graph as follows:

- The graph of an increasing function rises when read from left to right.
- The graph of a decreasing function falls when read from left to right.

*Many functions have some intervals on which they are increasing and other intervals on which they are decreasing. These intervals can often be identified from the graph.

Function Notation for the Average Rate of Change

Average rate of change of $Q = f(t)$ over the interval $a \le t \le b$ = $\dfrac{\Delta Q}{\Delta t} = \dfrac{f(b) - f(a)}{b - a}$.

Chapter 1.2 – *Example on pg. 10 in Text.*

Annual sales of VCR's and DVD's in millions of dollars

Year	1998	1999	2000	2001	2002	2003
VCR sales *(millions)*	2409	2333	1869	1058	826	407
DVD sales *(millions)*	421	1099	1717	2097	2427	3050

Average rate of change of VCR sales
from 1998 to 2003

Average rate of change of DVD sales
from 1998 to 2003

Chapter 1.2 – *Example 4 on pg. 14 in Text*

Calculate the average rate of change of the function $f(x) = x^2$ between:

a. $x = 1$ and $x = 3$

b. $x = -2$ and $x = 1$

Show your results on a graph.

c. Also calculate the average rate of change between $(a, f(a))$ and $(b, f(b))$.

Review ***Example 2 on pg. 12*** and ***Example 3 on pg. 13 in Text.***

Section 1.3 - *Linear Functions*

Constant Rate of Change
For many functions, the average rate of change is different on different intervals. In this section we consider functions which have the same average rate of change on every interval. Such a function has a graph which is a line and is called *linear*.

Population Growth
Mathematical models of population growth are used by city planners to project the growth of towns and states. Biologists model the growth of animal populations and physicians model the spread of an infection in the bloodstream. One possible model, a linear model, assumes that a population changes as the same average rate on every time interval.

Any linear function has the same average rate of change over every interval. Thus, we talk about the rate of change of a linear function. In general:

- A **linear function** has a constant rate of change.
- The graph of any linear function is a straight line.

A General Formula for the Family of Linear Functions

If $y = f(x)$ is a linear function, then for some constants *b* and *m*:

$$y = b + mx.$$

$$\text{Output} = \text{Initial value} + \text{Rate of change} \times \text{Input}$$

- *m* is called the **slope**, and gives the rate of change of *y* with respect to *x*. Thus,

$$m = \frac{\Delta y}{\Delta x}.$$

If (x_0, y_0) and (x_1, y_1) are any two distinct points on the graph of *f*, then

$$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}.$$

- *b* is called the **vertical intercept,** or **y-intercept**, and gives the value of *y* for $x = 0$. In mathematical models, *b* typically represents and initial, or starting, value of the output.

Every linear function can be written in the form *y* = *b* + *mx*. Different linear functions have different values for *m* and *b*. These constants are known as *parameters*. Parameters can be used to compare linear functions.

Not all graphs that look like lines represent linear functions:
The graph of any linear function is a line. However, a function's graph can look like a line without actually being one.

Chapter 1.3 – *Example 1 on pg. 18 in Text*

A town of 30,000 people grows by 2000 people every year. Since the population, *P*, is growing at a constant rate of 2000 people per year, *P* is a linear function of time, *t*, in years.

a. What is the average rate of change of P over every time interval?

b. Make a table that gives the town's population every five years over a 20-year period. Graph the population.

c. Find a formula for *P* as a function of *t*.

Chapter 1.3 – *Example 2 on pg. 19 in Text*

A small business spends $20,000 on new computer equipment and, for tax purposes, chooses to depreciate it to $0 at a constant rate over a five-year period.

a. Make a table and a graph showing the value of the equipment over the five-year period.

b. Give a formula for the value as a function of time.

Chapter 1.3 – *Example 4 on pg. 21 in Text*

Could either of the functions *p* and *q* be linear?

x	50	55	60	65	70
$p(x)$	0.10	0.11	0.12	0.13	0.14
$q(x)$	0.01	0.03	0.06	0.14	0.15

Chapter 1.3 – *Example 5 on pg. 22 in Text*

The former Republic of Yugoslavia exported cars called Yugos to the US between 1985 and 1989. The car is now a collector's item. The table below gives the quantity of Yugos sold, *Q*, and the price, *p*, for each year from 1985 to 1988.

a. Using the table explain why *Q* could be a linear function of *p*.

b. What does the rate of change of this function tell you about Yugos?

Year	Price in $, *p*	Number sold, *Q*
1985	3990	49,000
1986	4110	43,000
1987	4200	38,500
1988	4330	32,000

Section 1.4 - *Formulas for Linear Functions*

To find a formula for a linear function we find the values for the slope, m, and the vertical intercept, b in the formula $y = b + mx$.

Finding a Formula for a Linear Function from a Table of Data
If a table of data represents a linear function, we first calculate m and then determine b.

Finding a Formula for a Linear Function from a Graph
We can calculate the slope, m, of a linear function using two points on its graph. Having found m we can use either of the points to calculate b, the vertical intercept.

Finding a Formula for a Linear Function from a Verbal Description

Alternative Forms for the Equation of a Line

The following equations represent lines:

- The *slope-intercept form* is: $y = b + mx$ where m is the slope and b is the y-intercept
- The *point-slope form* is $y - y_0 = m(x - x_0)$ where m is the slope and (x_0, y_0) is a point on the line.
- The *standard form* is $Ax + By + C = 0$ where A, B, and C are constants.

If we know the slope of a line and the coordinates of a point on the line, it is often convenient to use the point-slope form of the equation.

Chapter 1.4 – *Example 1 on pg. 27 in Text*

A grapefruit is thrown into the air. Its velocity, v, is a linear function of t, the time since it was thrown. A positive velocity indicates the grapefruit is rising and a negative velocity indicates it is falling.

a. Check that the data in the table corresponds to a linear function.

b. Find a formula for v in terms of t.

c. What does the rate of change, m tell us about the grapefruit?

Chapter 1.4 – *Example 3 on pg. 30 in Text*

We have $24 to spend on soda and chips for a party. A six-pack of soda costs $3 and a bag of chips costs $2. The number of six-packs we can afford, *y*, is a function of the number of bags of chips we decide to buy, *x*.

a. Find an equation relating *x* and *y*.

b. Graph the equation. Interpret the intercepts and the slope in the context of the party.

Chapter 1.4 – *Problem on pg. 33 in Text*

In a college meal plan you pay a membership fee; then all your meals are at a fixed price per meal.

a. If 30 meals cost $152.50 and 60 meals cost $250, find the membership fee and price per meal.

b. Write a formula for the cost of a meal plan *C*, in terms of the number of meals, *n*.

c. Find the cost for 50 meals.

d. Find *n* in terms of *C*.

e. Use part d. to determine the maximum number of meals you can buy on a budget of $300.

Example (like homework #23)

Find a formula for the linear function *f* if *f* has $f(-1)=5$ and $f(2)=-1$

Review ***Example 2 on pg. 28 in Text.***

Section 1.5 - *Geometric Properties of Linear Functions*

Interpreting the Parameters of a Linear Function
The slope-intercept form for a linear function is $y = b + mx$, where b is the y-intercept and m is the slope. The parameters m and b can be used to compare linear functions.

The Effect of Parameters on the Graph of a Linear Function

Let $y = b + mx$. Then the graph of y against x is a line.

- The y-intercept, b, tells us where the line crosses the y-axis.
- If the slope, m, is positive, the line climbs from left to right. If the slope is negative, the line falls from left to right.
- The slope, m, tells us how fast the line is climbing or falling.
- The larger the magnitude of m, (either positive or negative), the steeper the graph of f.

Intersection of Two Lines
To find the point at which two lines intersect, notice that the (x, y)-coordinates of such a point must satisfy the equations for both lines. Thus, in order to find the point of intersection algebraically, solve the equations simultaneously. If linear functions are modeling real quantities, their points of intersection often have practical significance.

Equations of Horizontal and Vertical Lines

For any constant k:

- The graph of the equation $y = k$ is a horizontal line and its slope is **zero**.
  $y = b + 0 \cdot x$
- The graph of the equation $x = k$ is a vertical line and its slope is **undefined**.

Slopes of Parallel and Perpendicular Lines

Let l_1 and l_2 be two lines having slopes m_1 and m_2 respectively. Then:

- These lines are parallel if and only if $m_1 = m_2$.
- These lines are perpendicular if and only if $m_1 = -\frac{1}{m_2}$.

Chapter 1.5 – *Example 1 on pg. 35 in Text*
With time *t*, in years the populations of three towns are given by the following formulas:

$$P_A = 20,000 + 1600t$$

$$P_B = 50,000 - 300t$$

$$P_C = 650t + 45,000$$

$$P_D = 10,000(1.09)^t$$

a. Which populations are represented by linear functions?

b. Describe in words what each linear model tells you about the towns' population. Which town starts out with the most people? Which town is growing fastest?

c. When do towns A and B have the same populations?

Chapter 1.5 – *Example 3 on pg. 37 in Text*
The cost in dollars of renting a car for a day from three different rental agencies and driving it *d* miles is given by the following functions:

$$C_1 = 50 + 0.10d \qquad C_2 = 30 + 0.20d \qquad C_3 = 0.50d$$

a. Describe in words the daily rental arrangements made by each of these three agencies.

b. Which agency is the cheapest?

c. Using the graph, when will $C_1 = C_2$?

d. When is C_1 the most expensive?

Section 1.6 - *Fitting Linear Functions to Data*

When real data are collected in the laboratory or the field, they are often subject to experimental error. Even if there is an underlying linear relationship between two quantities, real data may not fit this relationship perfectly. However, even if a data set does not perfectly conform to a linear function, we may still be able to use a linear function to help us analyze the data.

Fitting the best line to a set of data is called *linear regression*. One way to fir a line is to draw a line "by eye". Alternatively, many computer programs and calculators compute regression lines.

Interpolation and Extrapolation

In general, interpolation tends to be more reliable than extrapolation because we are making a prediction on an interval we already know something about instead of making a prediction beyond the limits of our knowledge.

Correlation

When a computer or calculator calculates a regression line, it also gives a *correlation coefficient*, *r*. This number lies between -1 and +1 and measures how well a particular regression line fits the data. If $r = 1$, the data lie exactly on a line of positive slope. If $r = -1$, the data lie exactly on a line of negative slope. If *r* is close to 0, the data may be completely scattered, or there may be a non-linear relationship between the variables.

The Difference between Relation, Correlation, and Causation

It is important to understand that a high correlation (positive or negative) between to quantities does *not* imply causation. Also, a correlation of 0 does not imply that there is no relationship between *x* and *y*. A correlation of $r = 0$ usually implies there is no linear relationship between *x* and *y*, but this does not mean there is no relationship at all.

Chapter 1.6 – *Fitting Linear Functions to Data* *(NOT in Textbook)*

Example

Objective: Use real data, enter into the graphing calculator and have the calculator produce an equation that best fits the data. We will use this data to predict future events.

Data: The following data was collected by seismologists Allan Lindh and William Bakun who have studied a section of the San Andreas Fault near Parkfield, CA. Six quakes of magnitude 6 or better have hit that area. The following chart summarizes the data:

QUAKE #	1	2	3	4	5	6
YEAR	1857	1880	1902	1925	1945	1967

1. Put this information into the calculator.

2. Find the equation *(least square line)* that best represents *(fits)* this data using the calculator.

3. What is the correlation coefficient, *r* ?

4. Interpret the correlation between the earthquake number and the year it happened.

5. Sketch a graph using the calculator, and then draw a scatter plot below.

6. For this data predict when (what year) ***EARTHQUAKE #7*** will hit this area.