Section 5.1 - *Factoring Polynomial and Trigonometric Expressions*

The process of factoring undoes the process of multiplication.

If we change a product into a sum, we are multiplying.
$(x+5)(x+1) \xrightarrow{\text{multiplying}} x^2+6x+5$

If we change a sum into a product, we are factoring.
$x^2+6x+5 \xrightarrow{\text{factoring}} (x+5)(x+1)$

A **polynomial** is an algebraic expression that may be written as a finite sum of terms that contain only nonnegative integer exponents on the variables.

Factoring Out the GCF (Greatest Common Factor)
Progress Check 1 *Factor completely.*
a. $8y^3+4y^2$ b. $2x(x+3)+5(x+3)$

Progress Check 3 *Factor completely.*
a. $6s^2-9s$ b. $6\sin^2\theta-9\sin\theta$

Factoring by Grouping
Progress Check 4 *Factor by grouping.*
a. $2x^2+4x+3x+6$ b. $x^3-6x^2-6x+36$

Factoring Trinomials (Leading Coefficient 1)
Progress Check 5 *Factor completely* x^2-x-20.

Factoring Trinomials (Leading Coefficient Not 1) Using the ac Method
Progress Check 6 *Factor completely* $4t^2-4t-15$. (part b is the same except with trig functions)

Progress Check 7 *Factor completely* $4c^2-4c+1$. (part a is the same except with trig functions)

Combining Factoring Methods
Progress Check 9 *Factor completely* $6y^4-15y^3-9y^2$.

Factoring Test for Trinomials
A trinomial of the form ax^2+bx+c, where *a*, *b*, and *c* are integers, is factorable into binomial factors with integer coefficients if and only if b^2-4ac is a perfect square.
Progress Check 10 *Calculate* b^2-4ac *and determine if* $9x^2-9x-4$ *can be factored into binomial factors with integer coefficients.*

Section 5.2 - *Special Factoring Models and a Factoring Strategy*

Fill in the blank columns below:

PERFECT SQUARES				***PERFECT CUBES***			
	Perfect Square Numbers		**Perfect Square Variables**		**Perfect Cube Numbers**		**Perfect Cube Variables**
$1^2 =$		$\left(x^1\right)^2 =$		$1^3 =$		$\left(x^1\right)^3 =$	
$2^2 =$		$\left(x^2\right)^2 =$		$2^3 =$		$\left(x^2\right)^3 =$	
$3^2 =$		$\left(x^3\right)^2 =$		$3^3 =$		$\left(x^3\right)^3 =$	
$4^2 =$		$\left(x^4\right)^2 =$		$4^3 =$		$\left(x^4\right)^3 =$	
$5^2 =$		$\left(x^5\right)^2 =$		$5^3 =$		$\left(x^5\right)^3 =$	
$6^2 =$		$\left(x^6\right)^2 =$		$6^3 =$		$\left(x^6\right)^3 =$	
$7^2 =$		$\left(x^7\right)^2 =$		$7^3 =$		$\left(x^7\right)^3 =$	
$8^2 =$		$\left(x^8\right)^2 =$		$8^3 =$		$\left(x^8\right)^3 =$	
$9^2 =$		$\left(x^9\right)^2 =$		$9^3 =$		$\left(x^9\right)^3 =$	
$10^2 =$		$\left(x^{10}\right)^2 =$		$10^3 =$		$\left(x^{10}\right)^3 =$	

Difference of Squares

An important factoring method comes from reversing the following special product.

$(a+b)(a-b) = a^2 - ab + ab - b^2 = a^2 - b^2$

Example: Multiply $(x+3)(x-3)$

Factoring Model for a Difference of Squares

$$a^2 - b^2 = (a+b)(a-b)$$

Progress Check 1 *Factor completely.*

a. $n^2 - 16$ b. $100x^2y^2 - 49z^2$

Progress Check 2 *Factor completely.*

b. $1 - 4s^2$

Sum or Difference of Two Cubes

Factoring Model for Sums and Differences of Cubes

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

Progress Check 4 *Factor completely.*

a. $8y^3-1$ b. x^9+27y^3

Progress Check 5 *Factor completely.*

a. $5x^3-40$ b. $16m^4-81n^4$

Guidelines for Factoring a Polynomial

1. Factor out a GCF if possible
2. Check for factorizations according to the number of terms the polynomial has.
   Two Terms: Look for difference of squares or sum/difference of cubes
   Three Terms: Look at the coefficient *a* in a trinomial of the form ax^2+bx+c, and factor from there as follows
   - If *a* = 1 try FOIL reversal
   - If *a* is not 1, use the *ac* method
   - Look for the special case of a perfect square trinomial

   Four Terms: Try factoring by grouping
3. Make sure that you can't factor again.

Progress Check 6 *Factor completely* $64-16t^2$.

Progress Check 8 *Factor completely* y^6-1.

Section 5.3 - *Solving Equations by Factoring*

Definition of Quadratic Equation

A **second-degree** or **quadratic equation** is an equation that can be written in the form

$$ax^2 + bx + c = 0,$$

where a, b, and c are real numbers, with $a \neq 0$.

Zero Product Principle

For any numbers a and b,

$ab = 0$ if and only if $a = 0$ or $b = 0$.

Factoring Method for Solving Quadratic Equations

1. If necessary, change the form of the equation so one side is 0.
2. Factor the nonzero side of the equation
3. Set each factor equal to zero and obtain the solution(s) by solving the resulting equations
4. Check each solution by substituting it in the original equation.

Progress Check 1 *Solve* $3x^2 = -21x$ *using the factoring method.*

To Find Intercepts

1. To find x-intercepts, which have the form $(a, 0)$, let $y = 0$ and solve for x.
2. To find y-intercepts, which have the form $(0, b)$, let $x = 0$ and solve for y.

Progress Check 2 *Find the intercepts of the graph* $y = f(x) = x^2 + 2x - 8$.

Progress Check 3 *Solve* $x^2 + 4 = -4x$.

Progress Check 4 *Solve each equation.*

a. $(3x + 2)(2x - 3)(x - 1) = 0$ b. $x^3 = 4x$

Progress Check 6 The height (y) of a projectile that is shot directly up from the ground with an initial velocity of 80 ft/second is given by the formula $y = 80t - 16t^2$, where t is the time elapsed in seconds. For what value(s) of t is the projectile with 64 ft off the ground?

Section 5.4 - *Multiplication and Division of Fractions*

Algebraic fractions are the quotients of algebraic expression

Fraction Principles
Let *a, b, c, d,* and *k* be real numbers with *b, d,* and $k \neq 0$.

Equality of fractions $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$

Fundamental principle $\frac{ak}{bk} = \frac{a}{b}$

To Simplify Fractions
1. Factor completely the numerator and the denominator of the fraction
2. Divide out nonzero factors that are common to the numerator and the denominator according to the fundamental principle

Progress Check *Express in lowest terms.*

a. $\frac{c^2 - 8c + 16}{c^2 - 16}$ (part b is the same except with trig functions)

Progress Check 2 *Express in lowest terms.*

a. $\frac{4x}{8x^2 - 4x}$ b. $\frac{\left(x^2 - 5\right) - \left(c^2 - 5\right)}{x - c}$

Progress Check 3 *Express in lowest terms* $\frac{2 - 3x}{9x^2 - 4}$.

Progress Check 4 *Express in lowest terms the given products.*

a. $\frac{3c + 6}{c^2 + 4c} \cdot \frac{c^2 + 2c}{3c^2 - 12}$ (part b is the same except with trig functions)

Progress Check 5 *Multiply* $\frac{\left(x - 5\right)^2}{a^3 y^3} \cdot \frac{a^3 y^9}{\left(x - 5\right)^6}$. *Express answer in lowest terms.*

Progress Check 6 *Do each division and express the answer in lowest terms.*

a. $\frac{16}{x^4} \div \frac{12}{x^2}$ b. $\frac{\left(t + 5\right)^2}{4t} \div \frac{t^2 - 25}{12t^2}$ c. $\frac{y - 1}{x^2 + x} \div \frac{y - y^2}{x^2 + 4x + 3}$