

## 7.2 - Continuous Probability Distributions

### Characteristics of a Continuous Probability Distribution

1. The **probability** that the continuous random variable  $X$  will assume a value between two possible values,  $X = a$  and  $X = b$  of the variable **is equal to the area** under the density curve between the values  $X = a$  and  $X = b$ .
2. The **total area** (or **probability**) under the density curve is equal to **1**.
3. The **probability** that the continuous random variable  $X$  will assume a value between **any two possible values**  $X = a$  and  $X = b$  of the variable is **between 0 and 1**.

### Probability Statements Associated with a Continuous Probability Distribution

1. The probability of obtaining a specific value  $X = a$  of a continuous variable is zero. That is,  $P(X = a) = 0$ .
2. The probability that a continuous variable  $X$  assumes a value between  $a$  and  $b$ , written  $P(a < X < b)$ , and the probability that a continuous variable  $X$  assumes a value from  $a$  to  $b$ , written  $P(a \leq X \leq b)$ , are equal. That is,  $P(a < X < b) = P(a \leq X \leq b)$ .

## 7.3 - The Normal Distribution

- A normal distribution is a distribution that represents the values of a continuous variable.
- The importance of a normal distribution is that it serves as a *good model* in approximating many distributions of real-world phenomena.
- If a continuous variable is said to be approximately normal, then the normal distribution can be used to model the continuous variable.
- The graph of a normal distribution is called a normal curve which is a smooth symmetric bell-shaped curve with a single peak at the center.
- Examples of distributions that can be modeled or approximated by the normal distribution include IQ scores of individuals, adult weights, blood pressure of men and women, tire wear, the size of red blood cells, and the time required to get to work.

## 7.4 - Properties of a Normal Distribution

1. The normal curve is bell-shaped and has a single peak which is located at the center.
2. The mean, median, and mode all have the same numerical value and are located at the center of the distribution.
3. The normal curve is symmetric about the mean.
4. The data values in the normal distribution are clustered about the mean.
5. In theory, the normal curve extends infinitely in both directions, always getting closer to the horizontal axis but never touching it. As the normal curve extends out away from the mean and gets closer to the horizontal axis, the area in the tails of the normal curve is getting closer to zero.
6. The total area under the normal curve is 1 which can also be interpreted as probability. Thus, the area under the normal curve represents 100% of all the data values.

### The Standard Normal Curve

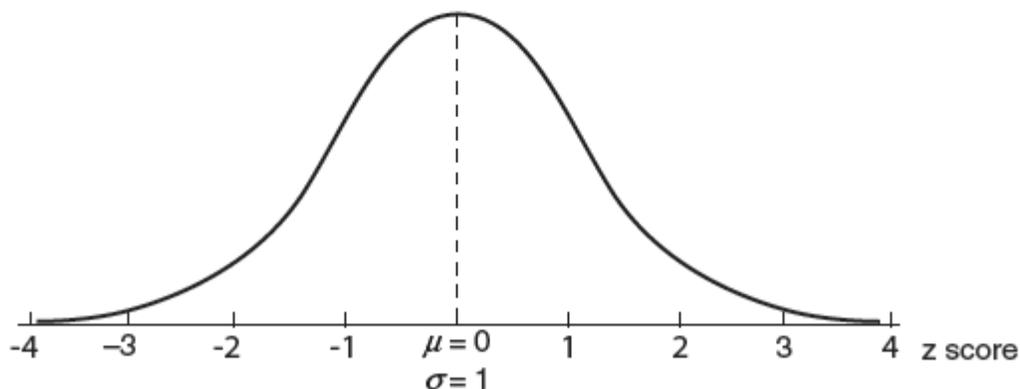
- For each continuous variable that can be approximated by a normal distribution with a particular mean and standard deviation, there is a normal curve having the same mean and standard deviation which serves as a model of this variable. Consequently, there is no single normal curve, but rather many normal curves which are referred to as a family of curves.
- Within this family of normal curves, there is only one normal distribution or curve for each combination of mean and standard deviation and the number of possible normal curves is infinite.
- The proportion of area under any normal curve between the mean and the data value which is  $N$  standard deviations away from the mean is the same for all normal curves. Consequently, when working with normal distribution applications it is possible only to have to work with one normal curve since each member of the family of normal distributions exhibits the same characteristics.
- Although there are an infinite number of normal curves, there is one special member of the normal distribution family that can be used to serve as the normal distribution model for any application. This special normal distribution is referred to as the **standard normal distribution**.
- The standard normal distribution has a mean equal to zero,  $\mu = 0$ , and a standard deviation equal to one,  $\sigma = 1$ . To utilize this standard normal distribution as a model for any normal distribution, it becomes necessary to convert any normal distribution to the standard normal distribution. This is accomplished by converting the values of the continuous variable to z scores.
- The z score formula helps to convert any normal distribution to the standard normal distribution.

The z score of a data value of  $x$  is equal to  $\frac{x - \mu}{\sigma}$ . Regardless of the values and units of any normal

continuous variable, the z score formula can be used to transform any normal distribution to the standard normal distribution with a mean of 0 and a standard deviation of 1.

- Since any normal distribution can be converted to the standard normal distribution, we only need to refer to one statistical table to find the area under any normal curve rather than an infinite number of tables for all the possible normal curves. This table is called the Standard Normal Curve Area Table.

The Standard Normal Distribution  
where  $\mu = 0$  and  $\sigma = 1$



### 7.5 - Using the Normal Curve Area Table

#### Procedure to Determine the Proportion of Area Under a Normal Curve to the Left of a z Score in Table II

1. Determine which page of Table II to use.
2. Find the row corresponding to the z score under the column labeled z. This row is determined by looking for the integer value and the value of the first decimal place of the z score.
3. Locate the column corresponding to the z score. To determine this column, look for the value of the second decimal place of the z score by moving across the top row of the table until you find the column pertaining to the second decimal place.
4. The entry found in the table where the row and the column determined in steps 2 and 3 intersect represents the proportion of the area to the left of the z score.

#### Procedure to Find the Proportion of Area to the Left of a z Score

Proportion of area to the left of z = Entry in Table II corresponding to z.

Using TI83/84 calculator:

*normalcdf* (lower bound, upper bound, mean, standard deviation)  
 $2^{\text{nd}}$  [DISTR] [2: normalcdf] ( [ -EE99] , [ z score] , 0 , 1 ) [ENTER]

#### Procedure to Find the Proportion of Area to the Right of a z Score

Proportion of area to the right of z = 1 – Proportion of area to the left of z.

Using TI83/84 calculator:

*normalcdf* (lower bound, upper bound, mean, standard deviation)  
 $2^{\text{nd}}$  [DISTR] [2: normalcdf] ( [ z score] , [EE99] , 0 , 1 ) [ENTER]

#### Procedure to Find the Proportion of Area Between Two z Scores

Proportion of area between two z scores = Proportion of area to the left of larger z score – Proportion of area to the left of the smaller z score.

Using TI83/84 calculator:

*normalcdf* (lower bound, upper bound, mean, standard deviation)  
 $2^{\text{nd}}$  [DISTR] [2: normalcdf] ( [ smaller z score] , [ larger z score] , 0 , 1 ) [ENTER]

#### Proportion of Area Associated to a Single z Score

Proportion of area for a particular z score equals zero.

**Example 7.1 on pg. 354** Find the proportion of area under **the normal curve**:

- a. To the left of  $z = -0.53$  (or Less than a z score of  $-0.53$ )

Using TABLE II finds area to the **left** of the z-score:

**Using your TI83/84 calculator:** *normalcdf* (– E 99 , z-score, mean , standard deviation )

- b. To the left of  $z = 2.56$  (or Less than a z score of 2.56)

**Example 7.2 on pg. 357** Find the proportion of area under **the normal curve**:

- a. To the right of  $z = -1.37$  (or greater than a z score of  $-1.37$ )

Using TABLE II finds area to the **left** of the z-score:

**Proportion of area to the right of z-score = 1 – Proportion of area to the left of z**

**Using your TI83/84 calculator:** *normalcdf* ( z-score, E 99 , mean , standard deviation )

- b. To the right of  $z = 0.67$  (or greater than a z score of 0.67)

**Example 7.3 on pg. 360** Find the proportion of area under **the normal curve**:

- a. Between  $z = 0$  and  $z = 1.5$

Using TABLE II finds area to the **left** of a z-score and we are asked for area **between two z-scores**:

Entry in table for **larger z-score**:

Entry in table for **smaller z-score**:

**Proportion between two z scores =**

Proportion of **area** to the left of **larger z score** – of **area** to the left of the **smaller z score**

**TI83/84 calculator:** *normalcdf* ( smaller z-score, larger z-score , mean , standard deviation )

- b. Between  $z = -1.96$  and  $z = 1.96$

- c. Between  $z = -1.25$  and  $z = 1.0$

**Procedure to Find the z-Score(s) Knowing the Area to the Left**

Using TI83/84 calculator:

*InvNorm (area to the left of the z-score, mean, standard deviation)*

*InvNorm (area to the left of the z-score, 0, 1 )*

**Example 7.5 on pg. 366** For a normal distribution, find the z score(s) that cut(s) off:

- a. The lowest 20% of area. (or cuts off the bottom 20 % or 0.20 of the z scores)

**What is the proportion of area to the left of the z-score we are looking for?**

- b. The highest of 10% of area (or cuts off the top 10 % or 0.10 of the z scores)

**What is the proportion of area to the left of the z-score we are looking for?**

- c. The middle 90% of the area (or of the z scores)

There will be **two** z-scores as your answer (one positive and one negative)

**What is the proportion of area to the left of the smaller z-score?**

**What is the proportion of area to the left of the smaller z-score?**

**\*There will always be two z-scores which are the same number but opposite signs when you cut off the “middle”.**

**Review Example 7.4 pg. 364**

### 7.6 - Applications of the Normal Distribution

**Example 7.6 on pg. 371** Assume the distribution of tire wear is approximately normal with:  $\mu = 48,000$ ;  $\sigma = 2,000$ . Find the **proportion** of tires which would be expected to wear:

- Less than 46,000 miles
- Greater than 49,000 miles
- Between 47,000 and 51,000 miles

**Using your TI83/84 calculator:**

*normalcdf(lower bound, upper bound, mean, standard deviation)*

- Less than 46,000 miles

**ANS:** The the proportion of tires that is expected to wear less than 46,000 miles is \_\_\_\_\_

- Greater than 49,000 miles

**ANS:** The proportion of tires that is expected to wear more than 49,000 is \_\_\_\_\_

- Between 47,000 and 51,000 miles

**ANS:** The proportion of tires that is expected to wear between 47,000 and 51,000 is \_\_\_\_\_

**Note:** If asked for the **percent** of tires which would be expected to wear between 47,000 and 51,000 the answer would be \_\_\_\_\_%

**Review Example 7.7 on pg. 374 in the text.**

**Example 7.8 on pg. 376** One thousand students took a standardized psychology exam. The results were approximately normal with  $\mu = 83$  and  $\sigma = 8$ . Find the **number** of students who scored:

- Less than 67

To find the **number** of students and not the proportion of area we must multiply the proportion with the **total number of students** that took the test.

(proportion of area) (total number of students) = number of students that scored less than 67

- Greater than 87

### 7.7 - Percentiles and Applications of Percentiles

- **Percentiles** are widely used to describe the position of a data value within a distribution.

The **percentile rank** of a data value  $X$  within a normal distribution is equal to the percent of area under the normal curve to the left of the data value  $X$ . **Percentile ranks are always expressed as a whole number.**

#### Procedure to Find Data Value (or Raw Score) that Cuts off Bottom $p\%$ Within a Normal Distribution

1. Change the percentage  $p$  to a proportion.
2. Using Table II, find the  $z$  score that has the area closest to this proportion. If there is a tie, choose the  $z$  score with the larger absolute value.
3. Substitute this  $z$  score into the raw score formula  $X = \mu + (z)\sigma$  to determine the data value that corresponds to this  $z$  score.

#### Procedure to Find Data Value (or Raw Score) that Cuts off Top $q\%$ Within a Normal Distribution

1. Calculate the percentage of data values below the raw score by subtracting  $q$  from 100:  $(100 - q)$ .
2. Change this percentage to a proportion.
3. Using Table II, find the  $z$  score that has the area closest to this proportion. If there is a tie, choose the  $z$  score with the larger absolute value.
4. Substitute this  $z$  score into the raw score formula  $X = \mu + (z)\sigma$  to determine the data value that corresponds to this  $z$  score.

**For percentile rank** – always use proportion of **area to the left** of the **data value** and **always** round to the nearest **whole number**.

#### Using your TI83/84 calculator for percentile rank:

$2^{nd}$  **DISTR** **2: normalcdf** (  $-EE99$  , **data value** ,  $\mu$  ,  $\sigma$  ) **ENTER**

*normalcdf* (  $-E 99$  , *data value* , *mean* , *standard deviation* )

**Example 7.9 on pg. 377** Matthew earned a grade of 87 on his history exam. If the grade in his class were normally distributed with  $\mu = 80$  and  $\sigma = 7$ , find Matthew's **percentile rank** on this exam.

#### Using your TI83/84 calculator for percentile rank:

*normalcdf* (  $-E 99$  , *data value* , *mean* , *standard deviation* )

normalcdf (  $-E 99$  , \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ ) = \_\_\_\_\_, now change the proportion this to a percent.

(proportion of area to the left) (100) = \_\_\_\_\_, now round to the nearest whole number...

**Percentile Rank** = \_\_\_\_\_

Notation for this is: \_\_\_\_\_

**Example 7.10 on pg. 377** Miss Brooke took two exams. Her grades and the class results are as follows:

<u>Exam</u>	<u>Her Grade</u>	<u>Class mean (<math>\mu</math>)</u>	<u>Class Standard Deviation (<math>\sigma</math>)</u>
English	83	80	6
Math	77	74	5

Use percentile ranks to determine on which test(s) Miss Brooke did better relative to her class

**For English:**

normalcdf ( \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ ) = \_\_\_\_\_ = \_\_\_\_\_ % = percentile rank is \_\_\_\_\_

The **percentile rank** of her English grade of 83 is \_\_\_\_\_, expressed as P = \_\_\_\_\_

**For Math:**

normalcdf ( \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ ) = \_\_\_\_\_ = \_\_\_\_\_ % = percentile rank is \_\_\_\_\_

The **percentile rank** of her Math grade of 77 is \_\_\_\_\_, expressed as P = \_\_\_\_\_

**Answer/Conclusion:** The percentile rank of her \_\_\_\_\_ grade is greater than the percentile rank of her \_\_\_\_\_ grade so relative to her class, Miss Brooke did **better on her \_\_\_\_\_ exam.**

**Example 7.11 on pg. 378** One of the entrance requirements at a local college is a percentile rank of at least 45 on the verbal SAT exam. Richard scored 470 on his verbal SAT exam. Does he meet the minimum requirement of this college, if the verbal SAT distribution is normally distributed with  $\mu = 500$  and  $\sigma = 100$  ?

**Example 7.13 on pg. 380** Consider a normal distribution with a mean of 75 and  $\sigma = 5$ . Find the data value(s) in the distribution that:

- Cut(s) off the top 25% of the data values.
- Represents the 30th percentile.
- Has a percentile rank of 55.
- Cut(s) off the middle 40% of the data values.

**Review Example 7.12 on pg. 379 in the text.**

