

### Chapter 2.1 - Functions, Relations and Ordered Pairs

#### Relations

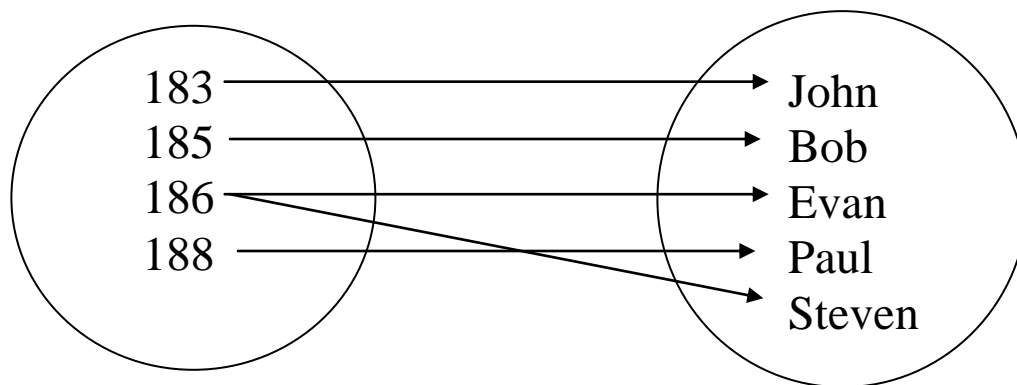
A **relation** is a set of ordered pairs. **Domain of a relation** is the set consisting of all the first elements of the ordered pairs. We call these elements the *input values*. **Range of a relation** is the set consisting of all the second elements of the ordered pairs. We call these elements the *output values*.

**Problem 1:** The relationship of height, in cm. and basketball players, names is a relation:

- a. A relation can be represented by a **table**:

Height (cm)	Player
183	John
185	Bob
186	Evan
186	Steven
188	Paul

- b. A relation can be represented by an arrow diagram:



- c. A relation can be represented by a set of **ordered pairs**:

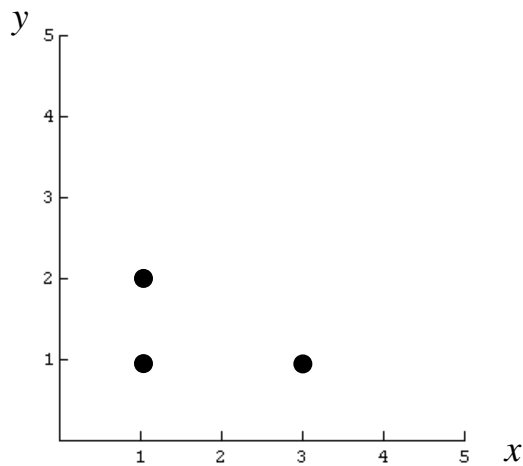
$$\{ (183, \text{John}), (185, \text{Bob}), (186, \text{Evan}), (186, \text{Steve}), (188, \text{Paul}) \}$$

**Problem 2:** The set of ordered pairs:  $\{(1,1), (1,2), (3,1)\}$

- a. This relation can be represented by a **table**:

$x$	$y$
1	1
1	2
3	1

- b. The ordered pairs can be represented on the Cartesian coordinate system or **graph**:



**Domain of a relation** is the set consisting of all the first elements of the ordered pairs, which are the *input values*.

Domain:  $\{1,3\}$

**Range of a relation** is the set consisting of all the second elements of the ordered pairs, which are the *output values*.

Range:  $\{1,2\}$

### Functions

A function is a special kind of relation. A **function** is a relation in which no two different ordered pairs have the same first component.

Relation

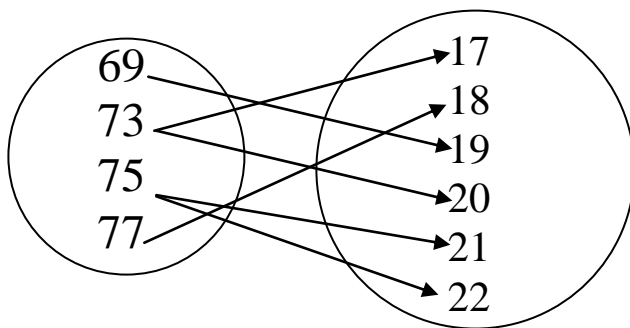
Height (cm)	Player
183	John
185	Bob
186	Evan
186	Steven
188	Paul

Relation that is a **Function**

Player	Height (cm)
John	183
Bob	185
Evan	186
Steve	186
Paul	188

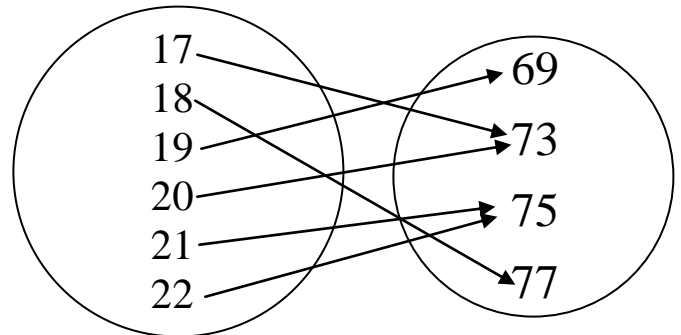
Relation

Temperature (°F)      Date in July



Relation that is a **Function**

Date in July      Temperature (°F)



Relation

{(10, 10), (10, 20), (10, 50)}

Relation that is a **Function**

{(10, 10), (20, 10), (50, 10)}

A **function** is a rule that assigns to each element  $x$  in a set  $X$  exactly one element  $y$  in a set  $Y$ . In this definition, the set  $X$  is called the **domain** of the function, and the set of all elements of  $Y$  that correspond to elements in the domain is called the **range** of the function.

- ▷ In goes an  $x$  value, then out comes **exactly one**  $y$  value.
- ▷ INPUT, OUTPUT
- ▷ INDEPENDENT VARIABLE, DEPENDENT VARIABLE
- ▷ DOMAIN, RANGE

$y$   
*output*  
*dependent variable*

“is a function of”  
“in terms of”

$x$   
*input*  
*independent variable*

**This rule can be in the form of a:**

- Table
- Graph
- List
- Formula
- Verbally

**Problem 3:** Finding your final grade from your numerical average. Which table below represents a function?

**TABLE A**

90 – 100	A
85 – 89	B+
80 – 84	B
75 – 79	C+
70 – 74	C
65 – 69	D+
60 – 64	D
Below 60	F

**TABLE B**

A	90 – 100
B+	85 – 89
B	80 – 84
C+	75 – 79
C	70 – 74
D+	65 – 69
D	60 – 64
F	Below 60

**Progress Check 1** Write a rule in equation form that describes the given function. Specify the domain of the function.

- a. Express the cost  $C$  of  $x$  gal of bottled water if the water costs \$1.35/gal (or per gallon)
- (a. Express the cost  $C$  of  $x$  gal of gasoline if the gas costs \$2.35/gal)
- b. Express the side length  $s$  of a square as a function of the area  $A$  of the square.

**Progress Check 3** Express the cost ( $c$ ) of a shipment of CD's as a function of the number ( $n$ ) of CD's ordered if the charge is \$9 per CD plus \$2.95 for shipping and handling. Describe the domain and range of this function.

**Progress Check 4** Determine the domain and range of each function.

a.  $y = \frac{1}{x+3}$

b.  $y = \sqrt{x-2}$

**Progress Check 5** Determine if the ordered pair is a solution of the equation.

a.  $x - 5y = 5$  (0, -1)

b.  $y = -2x + 5$  (1, 2)

**Progress Check 6 & 8** Determine if the given relation is a function and state the domain & range of the relation/function.

a.  $\{(65, D), (100, A), (43, F), (94, A)\}$

a.  $\{(-1, 1), (0, 0), (1, 1)\}$

b.  $\{(-1, 5), (-1, 6), (-1, 7)\}$

**Worksheet 2.1**

1. Is the following sets of ordered pairs a function? Explain & state the domain & range.

a.  $\{(1, 1), (1, 2), (3, 1)\}$

b.  $\{(4, 2), (2, 2), (3, 7)\}$

c.  $\{(90, A), (77, C), (89, B), (93, A)\}$

2. Is  $(2, 1)$  is a solution of the equation  $3x - y = 1$ .

3. If  $(b, -1)$  is a solution of the equation  $y = -2x + 9$ , find the value of  $b$ .

4. State the domain **and** range for the following functions:

a.  $y = \frac{x^2}{5}$

b.  $f(x) = \sqrt{x+7}$

c.  $f(x) = \sqrt{6-x}$

d.  $f(x) = \frac{1}{x-5}$

e.  $f(x) = \frac{1}{\sqrt{x-2}}$

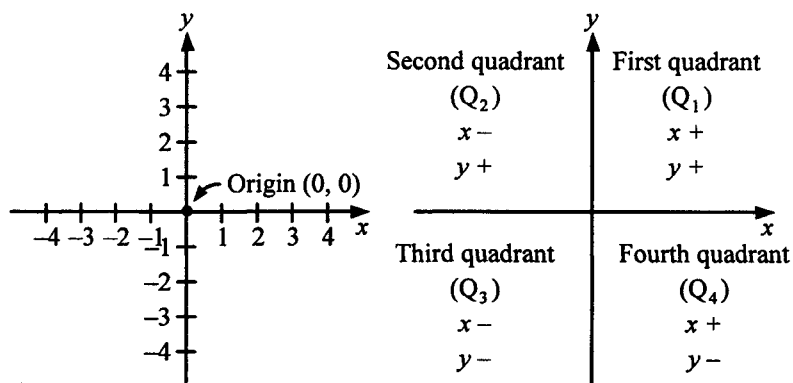
f.  $y = \frac{1}{x^2}$

5. Express the monthly cost ( $c$ ) for a checking account as a function of the number ( $n$ ) of checks serviced that month if the bank charges 10 cents per check plus a 75 cent maintenance charge.

a. Find a formula that defines the functional relationship between the two variables.

b. State the domain **and** range.

## Chapter 2.2 - Functions and Graphs



**Progress Check 1** Represent the ordered pairs  $(-2, 0)$ ,  $(-2, -4)$ ,  $(\frac{3}{2}, \pi)$  and  $(-\frac{5}{3}, \sqrt{5})$  as points in the Cartesian coordinate system.

### Graph

The graph of a relation is the set of all points in a coordinate system that correspond to ordered pairs in the relation.

**Progress Check 2** Graph the equation  $y = 2x - 1$ . Use a graphing calculator to check.

### Linear Equations in Two Variables

A **linear equation in two variables** is an equation that can be written in the general form

$$Ax + By = C,$$

where  $A$ ,  $B$ , and  $C$  are real numbers with  $A$  and  $B$  not both zero. The graph of a linear equation in two variables is a straight line.

### Graphing a Linear Equation:

Methods of graphing a linear equation include:

- Making a list of ordered-pair solutions by making a table of values for  $x$  and  $y$ .
- Finding the  $x$ - and  $y$ - intercepts
- Using a graphing calculator or graphing software

### To Find Intercepts

To find the  $x$ -intercept  $(a, 0)$ , let  $y = 0$  and solve for  $x$ .

To find the  $y$ -intercept  $(0, b)$ , let  $x = 0$  and solve for  $y$ .

**Progress Check 3** Graph the equation  $y = -x - 1$  by using the  $x$ - and  $y$ - intercepts. Use a graphing calculator to check.

**Progress Check 4** Graph the equation  $x - 2y = 4$  by using the  $x$ - and  $y$ - intercepts. Use a graphing calculator to check.

### Graphing Horizontal and Vertical Lines

**To Graph**  $x = a$  and  $x = b$

1. The graph of the linear equation  $x = a$  is a vertical line that contains the point  $(a, 0)$ . If  $a = 0$ , then the equation is  $x = 0$ , and the line is the  $y$ -axis.
2. The graph of the linear equation  $y = b$  is a horizontal line that contains the point  $(0, b)$ . If  $b = 0$ , then the equation is  $y = 0$ , and the line is the  $x$ -axis.

**Progress Check 5** Graph the equation  $y = -1$ .

**Progress Check 6** Graph the equation  $x = 3$ .

### Graphing a Parabola by Plotting Points

A function written in the form  $y = ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are real numbers with  $a \neq 0$ , is a parabola.

**Progress Check 7** Graph the equation  $y = x^2 - 4$  by plotting points. Use a graphing calculator to check.

**Progress Check 8** Graph the equation  $y = (x - 2)^2$  using a graphing calculator.

**Progress Check 9** Graph the equation  $y = 60 - x$  using the following viewing windows. Which one shows a complete graph?

- a.  $x\text{Min} = -10$   
 $x\text{Max} = 70$   
 $x\text{Scl} = 10$   
 $y\text{Min} = -10$   
 $y\text{Max} = 70$   
 $y\text{Scl} = 10$

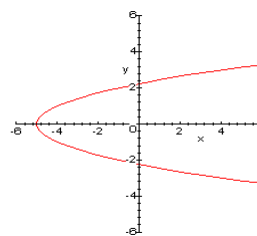
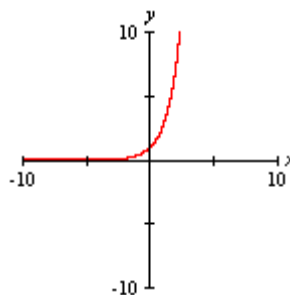
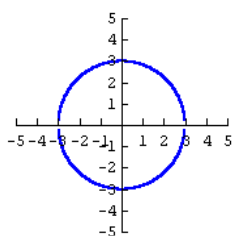
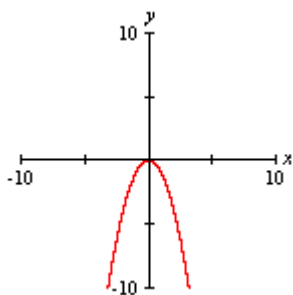
- b.  $x\text{Min} = -70$   
 $x\text{Max} = 10$   
 $x\text{Scl} = 10$   
 $y\text{Min} = -70$   
 $y\text{Max} = 10$   
 $y\text{Scl} = 10$

**Progress Check 10** Graph the equation  $y = 3x^2 + 8x$  and estimate the coordinates of the lowest point on the graph to the nearest tenth.

It is easy to recognize the graph of a function because none of its points can have the same x-coordinate. No point in the graph of a function can be directly above any other point. This feature is summarized by the vertical line test.

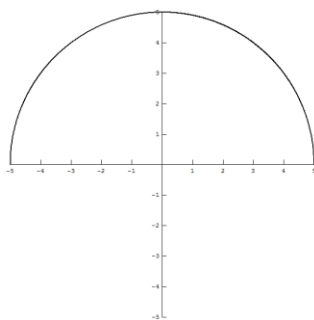
**Vertical Line Test**  
 Imagine a vertical line sweeping across a graph. If the vertical line at any position intersects the graph in more than one point, then the graph is not the graph of a function.

**Problem:** Which of the following graphs represent a function? (See Progress Check 11 and Example 11)



**Finding Domain and Range Graphically**

**Example 12** Find the domain and range of the relation in the figure below. Is this relation a function? (Do Progress Check 12)



**Finding Domain and Range Algebraically and Graphically**

**Progress Check 13** Determine the domain and range of  $y = -\sqrt{18-x}$ . Use both algebraic and geometric methods.

## Chapter 2.3 – Variation

In many scientific laws the functional relationship between variables is stated in the language of variation. The statement “ $y$  varies *directly* as  $x$ ” means that there is some nonzero number  $k$  such that  $y = kx$ . The constant  $k$  is called the **variation constant**. In some applications that relationship  $y = kx$  is also described by saying that  $y$  is **proportional** to  $x$ , and that  $k$  is the **constant of proportionality**.

### Variation

“ $y$  varies directly as  $x$ ” use  $y = kx$  or “ $y$  is proportional to  $x$ ”

“ $y$  varies directly as  $x^2$ ” use  $y = kx^2$

“ $y$  varies directly as  $x^3$ ” use  $y = kx^3$

**Progress Check 1** Write a variation equation for the given relation and determine the value of the variation constant if it is known.

- The circumference  $C$  of a circle varies directly as the diameter  $d$ .
- The property tax  $T$  on a house varies directly as the assessed value  $v$  of the house.

### Finding and Using $k$ in a Direct Variation Relation

**Progress Check 2** If  $y$  varies directly as  $x$ , and  $y = 3$  when  $x = 4$ , write  $y$  as a function of  $x$ . Find  $y$  when  $x = 12$ .

**Problem:** If  $y$  varies directly as  $x^3$ , and  $y = 5$  when  $x = 3$ , write  $y$  as a function of  $x$ . Find  $y$  when  $x = 2$ .

**Progress Check 3** The weekly earnings of a part-time employee vary directly as the number of hours worked. For an employee who makes \$111.60 for 18 hours of work, what are the earnings for 20 hours of work?

**Progress Check 4** The weight of an object on the moon is proportional to its weight of the object on Earth. An object that weighs 204 pounds on Earth weighs 34 pounds on the moon.

- Express the moon weight as a function of the Earth weight using an equation of variation, and find the value of the constant of proportionality.
- Graph the function from part **a** and give its domain.
- Using the equation from part **a** of the graph from part **b**, determine what a person who weighs 138 pounds on Earth will weigh on the moon?

In some relationships one variable decreases as another increases. If this happens in such a way that the product of the two variables is constant, then we say that the variables **vary inversely**, or that one is **inversely proportional** to the other. The statement “y varies inversely as x” means that there is some nonzero number  $k$  (the variation constant) such that  $xy = k$  or  $y = \frac{k}{x}$ .

### Finding and Using $k$ in an Inverse Variation Relation

#### Variation

“y varies inversely as or with x” use  $y = \frac{k}{x}$       “y is inversely proportional to x” use  $y = \frac{k}{x}$

“y varies inversely as  $x^2$ ” use  $y = \frac{k}{x^2}$

“y varies inversely as  $x^3$ ” use  $y = \frac{k}{x^3}$

**Progress Check 5** If  $y$  varies inversely with  $x$ , and  $y = 13$  when  $x = 5$ , write  $y$  as a function of  $x$ . Find  $y$  when  $x = 1$ .

**Problem:** If  $y$  varies inversely as the square of  $x$ , and  $y = 2$  when  $x = 4$ , write  $y$  as a function of  $x$ . Find  $y$  when  $x = 8$ .

**Progress Check 6** The speed of a pulley is inversely proportional to the radius of the pulley. The speed of the larger of two pulleys, which has a 2 inch radius, is 45 rpm. What is the speed of a  $\frac{1}{2}$  inch pulley connected to it?

### Finding and Using $k$ in a Combined Variation Relation

#### Variation

“y varies directly as  $x$  and inversely as  $z$ ” use  $y = \frac{kx}{z}$

“y varies directly as  $x^2$  and inversely as  $z^3$ ” use  $y = \frac{kx^2}{z^3}$

“y varies directly as  $x^3$  and inversely as  $z^2$ ” use  $y = \frac{kx^3}{z^2}$

**Progress Check 6** If  $y$  varies directly as  $x^3$  and inversely as  $z^3$ , and  $y = 10$  when  $x = 2$  and  $z = 5$ , determine the value of  $y$  when  $x = 5$  and  $z = 2$ .

## Chapter 2.4 – Functional Notation and Piecewise Functions

### Functional Notation

To indicate the quantity  $y$  is function of  $x$ , we write:

$y$  is a function of  $x$

$y$  equals “ $f$  of  $x$ ”

$$y = f(x)$$

Thus applying the **rule**  $f$  to the **input** value  $x$ , gives **output** value,  $f(x)$  or  $y$

So, **output** =  $f(x)$  and/or **output** =  $y$  and **input** =  $x$

$y$  is the **dependent variable** and  $x$  is the **independent variable**.

$$\text{Output} = f(\text{Input})$$

$$\text{Dependent} = f(\text{Independent})$$

**Progress Check 1** If  $y = f(x) = 5x - x^2$ , find  $f(4)$ ,  $f(20)$ , and  $f(-5)$ .

### Interpreting Functional Notation

**Progress Check 2** If the value  $V$  of a particular work of art is given by the function

$$V = f(x) = 50,000(1.07)^x, \text{ where } x \text{ is the number of years since its purchase at } \$50,000, \text{ find } f(6).$$

Interpret the meaning of  $f(6)$  in the context of this example.

### Evaluating Two Functions

**Progress Check 3** If  $f(x) = 1 - x$  and  $g(x) = x^2 + 2$  find  $4f(-1) - 2g(3)$ .

### Finding a Difference Quotient

**Progress Check 5** Find the difference quotient for  $f(x) = x^2 + 3$  in simplest form.

### Piecewise Functions

A piecewise function is a function in which different rules apply for different intervals of domain values.

**Progress Check 6** Express the weekly earnings ( $e$ ) of a salesperson in terms of the cash amount ( $a$ ) of merchandise sold if the salesperson earns \$600 per week plus 8% commission on sales above \$10,000.

**Evaluating and Graphing a Piecewise Function**

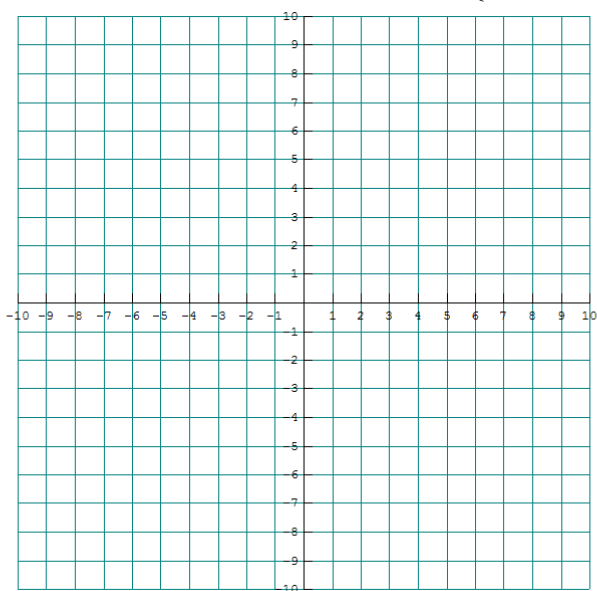
**Progress Check 7** If  $f(x) = \begin{cases} x-1 & \text{if } 0 \leq x < 5 \\ 0 & \text{if } x \geq 5 \end{cases}$  find

a.  $f(3)$

b.  $f(-3)$

c.  $f(5)$

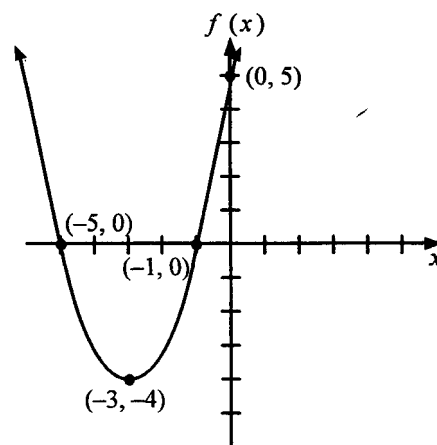
**Example 8** Graph the function defined as  $f(x) = \begin{cases} -1 & \text{if } x < 1 \\ x+2 & \text{if } x \geq 1 \end{cases}$



**Reading a Graph**

**Example 9** Consider the graph of  $y = f(x)$  in the figure below.

- What is the domain of  $f$ ?
- What is the range of  $f$ ?
- Determine  $f(0)$ .
- For what values of  $x$  does  $f(x) = 0$ ?
- For what values of  $x$  is  $f(x) < 0$ ?
- Solve  $f(x) > 0$ .

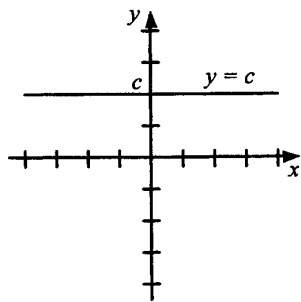


### Chapter 2.5 – Graphing Techniques

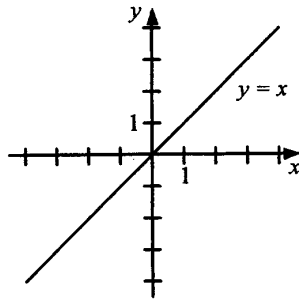
Many graphs can be sketched quickly by using the graph of variations of familiar functions by adjusting a known curve.

Three special functions are the following:

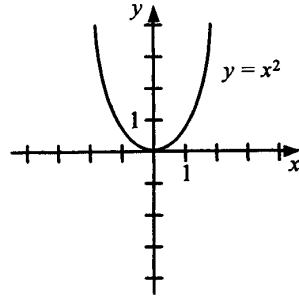
- Squaring Function:  $y = x^2$
- Square Root Function:  $y = \sqrt{x}$
- Absolute Value Function:  $y = |x|$



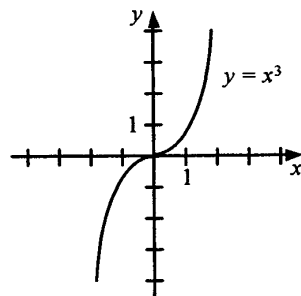
Constant Function



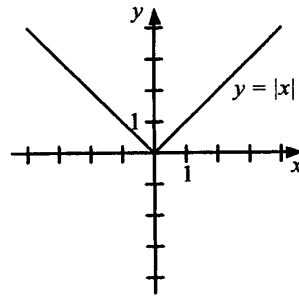
Identity Function



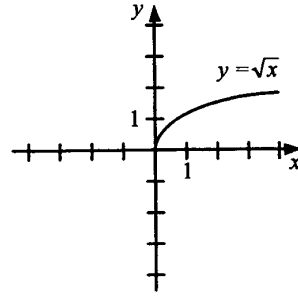
Squaring Function



Cubing Function



Absolute Value Function



Square Root Function

#### Vertical Translations

Let  $c$  be a positive constant.

1. The graph of  $y = f(x) + c$  is the graph of  $f$  shifted  $c$  units up.
2. The graph of  $y = f(x) - c$  is the graph of  $f$  shifted  $c$  units down.

**Progress Check 1** Use the graph of  $y = |x|$  to graph each function.

- a.  $y = |x| + 2$
- b.  $y = |x| - 4$

#### Horizontal Translations

Let  $c$  be a positive constant.

1. The graph of  $y = f(x + c)$  is the graph of  $f$  shifted  $c$  units to the left.
2. The graph of  $y = f(x - c)$  is the graph of  $f$  shifted  $c$  units to the right.

**Progress Check 3** Use the graph of  $y = |x|$  to graph each function.

- a.  $y = |x - 3|$
- b.  $y = |x + 1|$

**Problem:** Fill in each table using the equation given. What do you notice?

$$y = x^2$$

x	-2	-1	0	1	2	3
y						

$$y = (x+1)^2$$

x	-2	-1	0	1	2	3
y						

$$y = (x-1)^2$$

x	-2	-1	0	1	2	3
y						

**Progress Check 4** Graph  $y = (x-2)^2 + 1$ . Identify the vertex and the y-intercept of the graph.

### Reflecting, Stretching and Flattening

**To Graph**  $y = c \cdot f(x)$ :

**Reflecting** The graph of  $y = -f(x)$  is the graph of  $f$  reflected about the x-axis.

**Stretching** If  $c > 1$ , the graph of  $y = c \cdot f(x)$  is the graph of  $f$  stretched by a factor of  $c$ .

**Flattening** If  $0 < c < 1$ , the graph of  $y = c \cdot f(x)$  is the graph of  $f$  flattened out by a factor of  $c$ .

**Example 5** Graph each function.

a.  $y = -\sqrt{x}$

b.  $y = \frac{1}{4}x^2$

**Progress Check 5** Graph each function.

a.  $y = 3\sqrt{x}$

b.  $y = -x^2$

**Progress Check 6** Graph of  $y = 2 - |x|$  using the graph of  $y = |x|$ .

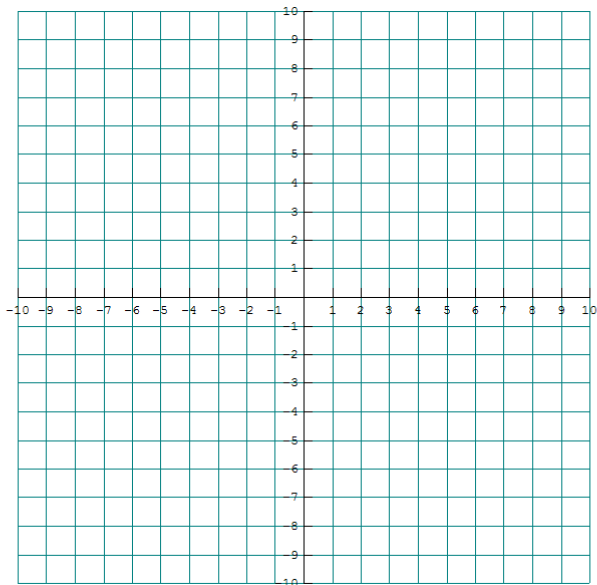
**Progress Check 7** Graph of  $y = 2 - |x-1|$  using the graph of  $y = |x|$ .

**Transformation Rules**

<b>Vertical Shifts:</b>	$y = f(x) + c$	raise the graph of $f$ $c$ <b>units UP</b>
	$c > 0$	
	$y = f(x) - c$	lower the graph of $f$ $c$ <b>units DOWN</b>
<b>Horizontal Shifts:</b>	$y = f(x + c)$	shift the graph of $f$ $c$ <b>units LEFT</b>
	$c > 0$	
	$y = f(x - c)$	shift the graph of $f$ $c$ <b>units RIGHT</b>
<b>Vertical Stretching:</b>	$c > 1$	$y = c \cdot f(x)$ <b>multiply</b> each $y$ value of $f$ by $c$
<b>Vertical Flattening:</b>	$0 < c < 1$	$y = c \cdot f(x)$ <b>multiply</b> each $y$ value of $f$ by $c$
<b>Vertical Reflection:</b>		$y = -f(x)$ reflect the graph of $f$ <b>about the <math>x</math>-axis</b>

**Practice Transformation Problem:**

- Sketch the graph of  $y = 6 - (x + 2)^2$ , using transformations on the graph  $f(x) = x^2$ . Label any intercepts and maximum or minimum point. *Explain the transformation.*



## Chapter 2.6 – Absolute Value Equations and Inequalities

The absolute value function:  $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

**Solution of  $|ax+b|=c$**

If  $|ax+b|=c$  and  $c > 0$ , then

$\Rightarrow$  Solve:  $ax+b=c$  or  $ax+b=-c$

$\Rightarrow$  Check your answers!

**Progress Check 1** Solve  $|x|=8$  using:

- a. a number line approach      b. a graphing approach      c. an algebraic approach

**Progress Check 2** Solve  $|14x-21|=63$ .

**Solution of  $|ax+b|=|cx+d|$**

If  $|ax+b|=|cx+d|$ , then

$\Rightarrow$  Solve:  $ax+b=cx+d$  or  $ax+b=-(cx+d)$

$\Rightarrow$  Check your answers!

**Progress Check 3** Solve  $|7x-8|=|9x+12|$ .

**Solution of  $|ax+b|<c$**

If  $|ax+b|<c$ , and  $c > 0$ , then

$\Rightarrow$  Solve:  $-c < ax+b < c$

$\Rightarrow$  Check your answers!

**Progress Check 6** Solve  $|5x+7|<12$ .

**Solution of  $|ax+b|>c$**

If  $|ax+b|>c$ , and  $c > 0$ , then

$\Rightarrow$  Solve:  $ax+b>c$  or  $ax+b<-c$

$\Rightarrow$  Check your answers!

**Progress Check 7** Solve  $|x-75|>5$ .

**Algebraic Approach** (for  $c > 0$ )**1. Solution of  $|ax + b| = c$** 

$\Rightarrow$  Solve:  $ax + b = c$  or  $ax + b = -c$

$\Rightarrow$  Check your answers!

**2. Solution of  $|ax + b| = |cx + d|$** 

$\Rightarrow$  Solve:  $ax + b = cx + d$  or  $ax + b = -(cx + d)$

$\Rightarrow$  Check your answers!

**3. Solution of  $|ax + b| < c$** 

$\Rightarrow$  Solve:  $-c < ax + b < c$

$\Rightarrow$  Check your answers!

**4. Solution of  $|ax + b| > c$** 

$\Rightarrow$  Solve:  $ax + b > c$  or  $ax + b < -c$

$\Rightarrow$  Check your answers!

**Graphing Approach**

- $y_1 = abs(ax + b)$
- $y_2 = c$
- find the point(s) of intersection

- $y_1 = abs(ax + b)$
- $y_2 = abs(cx + d)$
- find the point(s) of intersection

- $y_1 = abs(ax + b)$
- $y_2 = c$
- find the point(s) of intersection ( $x$  value)
- **answer** is the interval between the intersection points  $x$  values
- **Answer is what interval,  $x$  values, is the absolute function BELOW the horizontal line**
- $(\min x, \max x)$  of intersection points

- $y_1 = abs(ax + b)$
- $y_2 = c$
- find the point(s) of intersection ( $x$  value)
- **answer** is the interval to the left of the minimum  $x$  value and to the right of the maximum  $x$  value
- **Answer is what interval,  $x$  values, is the absolute function ABOVE the horizontal line**
- $(-\infty, \min x) \cup (\max x, \infty)$  of intersection points

## Chapter 2.7 – The Slope of a Line and the Distance Formula

The **slope** of a line measures its steepness or the degree to which it slants. The slope measures the change in the  $y$ -values with respect to the change in the  $x$ -values.

**Slope of a Line** If  $(x_1, y_1)$  and  $(x_2, y_2)$  are any two distinct points on a line  $x_1 \neq x_2$ , then the slope  $m$  of the line is:

$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

**Progress Check 1** Find the slope of the line through the given points.

- a.  $(1, -1)$  and  $(5, 3)$                       b.  $(1, -1)$  and  $(-2, 3)$

### Finding the Slope of Horizontal or Vertical Lines

**Progress Check 2** Find the slope of the line through the given points.

- a.  $(6, 1)$  and  $(0, 1)$                       b.  $(-2, -3)$  and  $(-2, 1)$

### Interpreting Slope as Steepness or Pitch

**Progress Check 3** Find the slope (pitch) of the wall of an A-frame ski lodge that rises 20 ft vertically through a horizontal distance of 15 ft.

**Progress Check 4** A printing shop charges \$50 for 500 copies of a flyer and \$58 for 700 copies. If the relation between the number of copies ( $x$ ) and the cost ( $y$ ) graphs as a line, calculate and interpret the slope.

### Parallel and Perpendicular Lines

- Two non-vertical lines are parallel if and only if their slopes are equal.
- Two non-vertical lines are perpendicular if and only if the product of their slopes is  $-1$ . The slope of one is the negative of the reciprocal of the slope of the other.

### Distance Formula

The distance  $d$  between  $(x_1, y_1)$  and  $(x_2, y_2)$  is  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .

**Progress Check 7** Find the distance between  $(-3, 2)$  and  $(-4, -2)$ .

**Progress Check 6** Find the distance between the given points.

- a.  $(3, 5)$  and  $(3, -3)$                       b.  $(-7, -1)$  and  $(0, -1)$

**Progress Check 8** Show that  $A(-2, 0)$ ,  $B(3, 5)$  and  $C(4, 2)$  are vertices of a right triangle using

- a. the distance formula                      b. the slope formula.

## Chapter 2.8 – Linear Functions

### Linear Functions

**Definition of a Linear Function**

A function of the form:

$$y = ax + b$$

where  $a$  and  $b$  are real numbers with  $a \neq 0$ , is called a **linear function**.

**Slope-Intercept Equation**

The graph of the equation:

$$y = mx + b$$

is a line with slope  $m$  and  $y$ -intercept  $(0, b)$

This equation is called the **slope-intercept** form of the equation of a line.

**Point-Slope Equation**

An equation of the line with slope  $m$  passing through  $(x_1, y_1)$  is:

$$y - y_1 = m(x - x_1)$$

This equation is called the **point-slope** form of the equation of a line.

**Progress Check 1** Find an equation the line through  $(-1, 0)$  with slope 3. Write the answer in the form  $y = mx + b$ .

**Progress Check 2** Find an equation the line that contains the points  $(-2, 2)$  and  $(3, -2)$ . Write the answer in the form  $y = mx + b$ .

**Progress Check 3** Find the slope and the  $y$ -intercept of the line defined by the following equations.

a.  $y = -2x + 3$

b.  $4x - 3y = 12$

**Progress Check 4** The slope of a line is  $-\frac{3}{4}$  and  $y$ -intercept is 5.

a. Find the equation of the line in slope-intercept form.

b. Graph the line.

**Progress Check 5** Find an equation the line that defines the linear function  $f$  if  $f(2) = 7$  and  $f(4) = -1$ . Write the answer in the form  $y = mx + b$ .

**Progress Check 6** During exercise a person's minimum target heart rate to have a training effect is a function of age. This relation is specified by a linear function, and the recommended minimum number of beats per minute is 126 at age 40 and 112 at age 60.

- Find the equation the line that defines this linear function.
- What is the minimum target rate for a 19-year-old?

**In any table of a linear function  $y = f(x)$ , a fixed change in  $x$  produces a constant difference between the corresponding  $y$ -values.**

**Example 7** The monthly cost  $y$  for  $x$  hours of usage of an "online" computer service company is shown in the following table.

Time (in hours), $x$	1	2	3	4	5
Cost (in dollars), $y$	14.75	19.55	24.35	29.15	33.95

See **Progress Check 7** it shows a table that is *not linear*.

**Progress Check 8** Determine if the graphs of  $2x + y = 1$  and  $2x + y = 3$  are distinct parallel lines.

**Progress Check 9** Find an equation for the line through the point  $(3, 4)$  that is

- parallel to  $x - 2y = 1$  and
- perpendicular to  $x - 2y = 1$ .