Step 1: Formulate the hypotheses.

Step 2: Determine the model (value) to test the null hypothesis.

Step 3: Formulate a decision rule.

Step 4: Analyze the sample data.

Step 5: State the conclusion.
A **Type I Error** is made when a true null hypothesis is incorrectly rejected.

A **Type II Error** is made when a false null hypothesis is not rejected.

<table>
<thead>
<tr>
<th>Conclusion:</th>
<th>Reality:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reject $H_0$</strong></td>
<td><strong>$H_0$ is True</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Type I Error</strong></td>
</tr>
<tr>
<td><strong>Fail to Reject $H_0$</strong></td>
<td><strong>Correct Decision</strong></td>
</tr>
</tbody>
</table>
THE SAMPLING DISTRIBUTION OF THE PROPORTION

The **sampling** distribution of the proportion is a probability distribution which lists all the possible values of the sample proportions of the same sample size selected from a population along with the probability associated with each value of the sample proportion.

*Ex:*

*Colors of the candy in bags of M&M’s*
The sample proportion is the ratio of the number of occurrences in the sample to the sample size. The sample proportion, denoted by $\hat{p}$ (read p-hat), can be expressed by the formula:

$$\hat{p} = \frac{x}{n}$$

where: $\hat{p} =$ sample proportion  
$x =$ number of occurrences in the sample  
n =$ sample size
THE SAMPLING DISTRIBUTION OF THE PROPORTION

\[ \hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4, \hat{p}_5, \ldots, \hat{p}_n \]
The Mean of the Sampling Distribution of the Proportion = mean of the population proportion:

\[ \mu_{\hat{p}} = p \]

Standard Error of the Proportion:

\[ SE \quad or \quad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} \]
Example: Symbolize the values

Last year 75% of adults in the USA believed a college degree is important. This year a poll selected 1,020 US adults and found that 794 believed a college degree is important. Can you conclude that the percent of adults in the US who believe a college degree is important has significantly increased this year?

Perform a hypothesis test at $\alpha = 5\%$. 
\[
\hat{p} = \frac{794}{1020} = 0.7784
\]
\[
Z_{\hat{p}} = \frac{0.7784 - 0.75}{0.0136} = 2.088
\]

\textit{given} \quad \alpha = 5\%

\[
\approx Z_c = 1.65
\]

\[
\hat{p}_c = Z_c \sigma_{\hat{p}} + \mu_{\hat{p}}
\]

\[
\hat{p}_c = 0.77244
\]
Reject Ho, accept Ha

normalcdf(0.7784, B, 0.75, 0.0136)

P-VALUE

0.0183882431
If the \( p\text{-value} < \alpha \) reject the null hypothesis, \( H_0 \).

What does this mean graphically?

(The exact location of the \( ev \) from the closest tail)
Interpreting $p$-Values

1. If the $p$-value is less than 0.01 (< 1%), the statistical result is **very significant**.

2. If the $p$-value is between 0.01 and 0.05 (1% - 5%), the statistical result is **significant**.

3. If the $p$-value is between 0.05 and 0.10 (5% - 10%), the statistical result is **marginally (or not) significant**.

4. If the $p$-value is greater than 0.10 (> 10%), the statistical result is **not significant**.
Note the type of $H_0$ value given

Is it in the form of a probability (percent) or a numerical value?

Therefore proportion or means use

\[
\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} \quad \text{or} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}
\]
Hypothesis Testing Check List

STATE (all that apply):
Ho, Ha, sample size, success ratio, failure ratio, conclusion worded statistically, probability of a Type I error, probability of a Type II error

SHADE:
1TT, 2TT, reject ion area(s) - indicate fail to reject

CALCULATE (show formulas and exact calculator functions used):
SE, critical value(s), Z score(s), p-value

LABEL:
mean, α, β, critical value(s), Z score(s)
experimental value (Use all symbols that apply)

INTERPRET:
Conclusion in English (relative to the problem), Type I error, Type II error, significance level, confidence level, p-value, p-value significance
Null hypothesis
Alternative hypothesis
Standard error
Alpha - significance level
Beta – confidence level
Sample size
Population size
Mean of the population
Mean of the sample means
Mean of the proportions

$Ho$
$Ha$
$SE$
$\alpha$
$\beta$
n
N
$\mu$
$\mu_{\bar{x}}$
$\mu_{\hat{p}}$
Notation Vocabulary List -2

- Standard deviation of the population: $\sigma$
- Standard deviation of the sampling distribution of means: $\sigma_x$
- Standard deviation of the sampling distribution of proportions: $\sigma_{\hat{p}}$
- Probability of success / proportion mean: $p, \hat{p}$
- Probability of failure: $q$
- Sample proportion: $\hat{p}$
Notation Vocabulary List -3

- Raw score in a population \( x \)
- Sample mean \( \bar{x} \)
- Critical raw score \( x_c \)
- Critical Z score of the sampling distribution of the means \( Z_c \)
- Critical Z score of the sampling distribution of the proportion \( Z_{\hat{p}} \)
A recent census report states 50% of U.S. families earn more than $20,000 per year. A sociologist believes this percent is too low. He randomly selects 100 families and determines that 64 have incomes of more than $20,000. The decision rule is to reject $H_0$ if the test statistic is more than 2.33.
Ho: \( p = 0.50 \)  
Ha: \( p > 0.50 \)  
n = 100  
\( p = 0.50 \)  
\( q = 0.50 \)  
mean proportion assumed  
\( \mu_\hat{p} = 0.50 \)

\( \alpha = 1\% * \) = Type I = significance level -English  
(reject the claim 50% earn more than $20,000, as an error)

\( \beta = 99\% * \) = Type II = confidence level -English  
(Fail to reject the claim 50% earn more than $20,000, as an error)

experimental value  \textbf{sample} = 64
Calculations

\[ \hat{p} = \frac{64}{100} = 0.64 \]

\[ \sigma_{\hat{p}} = \sqrt{\frac{(0.50)(0.50)}{100}} = 0.05 \]

\[ \begin{align*}
given Z_{c\hat{p}} &= \text{normalcdf}(A, 2.33) \\
&\approx 0.990096947 \\
&\approx \alpha = 0.01, \ 1TT \ right
\end{align*} \]

\[ Z_{\hat{p}} = \frac{0.64 - 0.50}{0.05} = 2.8 \]
Solution

\[ \text{SE} = \sigma_{\hat{p}} = 0.05 \]

\[ \mu_{\hat{p}} = 0.50 \]

\[ Z_{\hat{p}} = -2.8 \]

\[ Z_{c\hat{p}} = 2.33 \]

Locate experimental value = \( \hat{p} = 0.64 \)
conclusion statistically

Reject Ho, accept Ha

conclusion in English

reject the claim 50% earn more than $20,000,
accept the claim is higher than 50% .

\[
p\text{-value} = 0.00255
\]

\[
\text{normalcdf}(2.8, B) \quad .0025551906
\]

\[
p\text{-value interpretation} - \text{smallest } \alpha \text{ necessary to reverse the conclusion; results are very significant.}
\]
According to the latest figures unemployment is at 8.7%. The Congressman from the third district believes that the unemployment rate is lower in his district. To test his belief he interviews 200 residents of his district and finds 9 of them to be unemployed. Is the Congressman’s belief correct? Use $\alpha = 5\%$
Ho: $p = 0.087$  
Ha: $p < 0.087$

$n = 200$  
$p = 0.087$  
$q = 0.913$

mean proportion assumed $\mu_{\hat{p}} = 0.087$

$\alpha = 5\%$ = Type I = significance level - English
(reject the unemployment figures as an error)

$\beta = 95\%$ = Type II = confidence level - English
(Fail to reject the unemployment figures as an error)

experimental value sample $= 9$
\[ \hat{p} = \frac{9}{200} = 0.045 \]

\[ \sigma_{\hat{p}} = \sqrt{\frac{(0.087)(0.913)}{200}} = 0.019928 \]

\[ Z_{c\hat{p}} = \text{invNorm}(0.05) \approx -1.65 \text{ table} \]

\[ Z_{\hat{p}} = \frac{0.045 - 0.087}{0.019928} = -2.1075 \]
Solution

$Z_{\hat{p}} = -2.1075$

$Z_{c\hat{p}} = -1.65$

$SE = \sigma_{\hat{p}} = 0.019928$

$\hat{p} = 0.045$

$\mu_{\hat{p}} = 0.087$
conclusion **statistically**

*Reject Ho, accept Ha*

conclusion in **English**

reject the unemployment figures,
accept the claim the unemployment figures are lower.

\[ p\text{-value} = 0.01753/4 \]

\[ \text{normalcdf}(A,-2.1075) \]
\[ .0175370685 \]
\[ \text{normalcdf}(A,0.045,0.087,0.019928) \]
\[ .0175332886 \]

**p-value** interpretation — *smallest \( \alpha \) necessary to reverse the conclusion; results are **significant**.*
$
\text{SE} = \frac{\sigma}{x}, \frac{\sigma}{\hat{p}}$

$\mu_x$, $\mu_{\hat{p}}$

$Z_c$, $Z_{\hat{p}}$

$Z_{\hat{c}}$

Locate experimental value $= x, \hat{p}$
\[ \text{Locate experimental value} = \bar{x}, \hat{p} \]
General Design 2TT ≠

\[
\text{SE} = \frac{\sigma}{x}, \frac{\sigma}{\hat{p}}
\]

Locate experimental value = \( x, \hat{p} \)
conclusion statistically

conclusion in English

$p$-value

$p$-value interpretation

$p$-value significance
A psychiatrist wants to determine if a small amount of alcohol decreases the reaction time in adults. The mean reaction time for a specified test is 0.20 seconds. A sample of 64 people were given a small amount of alcohol and their reaction time averaged 0.17 seconds. Does this sample result support the psychiatrist’s hypothesis? Use $s = 0.08$ sec. and $\alpha = 1\%$. 
Ho: $\mu = 0.20$ sec  \hspace{1cm} Ha: $\mu < 0.20$ sec

$n = 100$

mean assumed $\mu_x = 0.20$ sec

$\alpha = 1\%$ = Type I = significance level – English: (reject the population statement of 0.20 sec. as an error)

$\beta = 99\%$ = Type II = confidence level – English: (fail to reject the population statement of 0.20 sec. as an error)

Experimental value $\bar{x} = 0.17$ sec
Calculations

\[
\frac{s}{\bar{x}} = \frac{0.08}{\sqrt{64}} = 0.01
\]

\[
Z_c = \text{invNorm}(0.01) \approx -2.33
\]

\[
Z = \frac{0.17 - 0.20}{0.01} = -3
\]
Solution

\[ Z_{\bar{x}} = -3 \]

\[ Z_c = -2.33 \]

\[ \mu_{\bar{x}} = 0.20 \text{ sec} \]

\[ SE = 0.01 \]

\[ \bar{x} = 0.17 \text{ sec} \]
conclusion statistically

Reject Ho, accept Ha

conclusion in English

reject the population statement of 0.20 sec.
accept the psychiatrist’s claim of < 0.20 sec.

\[
p-value = 0.00134
\]

\[
\begin{align*}
normalcdf(A, -3) & = 0.0013499672 \\
normalcdf(A, 0.17, 0.20, 0.01) & = 0.0013499672
\end{align*}
\]

\[p-value\text{ interpretation} – \text{smallest } \alpha \text{ necessary to reverse the conclusion; results are very significant.}\]
A drug company claims to have developed a new antibiotic which is more effective against type A bacteria than its current product. The current product is 90% effective. To test its claim a random sample of 400 individuals who were infected with this bacteria are treated with the new antibiotic and 380 recover. Do these results support the drug company’s claim? $\alpha = 1\%$. 
Ho: $p = 0.90$  \hspace{1cm} Ha: $p > 0.90$

$n = 400$  \hspace{1cm} $p = 0.90$  \hspace{1cm} $q = 0.10$

mean proportion assumed  $\mu_p = 0.90$

$\alpha = 1\%$  \hspace{1cm} = Type I = significance level - English

(reject claim the new product is not more effective - status quo, as an error)

$\beta = 99\%$  \hspace{1cm} = Type II = confidence level - English

(Fail to reject claim the new product is not more effective, as an error)

experimental value  sample  = 380
\[ \hat{p} = \frac{380}{400} = 0.95 \]

\[ \sigma_{\hat{p}} = \sqrt{\frac{(0.90)(0.10)}{400}} = 0.015 \]

\[ Z_{c\hat{p}} = \text{invNorm}(0.99) \approx 2.33 \]

\[ Z_{\hat{p}} = \frac{0.95 - 0.90}{0.015} = 3.33 \]
SE = $\sigma_{\hat{p}} = 0.015$

$\mu_{\hat{p}} = 0.90$

$Z_{\hat{p}} = 3.33$

$Z_{c\hat{p}} = 2.33$

Locate experimental value $= \hat{p} = 0.95$
conclusion statistically

rejected Ho, accept Ha

conclusion in English

reject claim the new product is not more effective
accept the drug company claim the new product is more effective.

p-value = 0.000434

p-value interpretation – smallest $\alpha$ necessary to reverse the conclusion; results are very significant.
Wikipedia, a free online encyclopedia, states that in 2006, the average US woman is 25 years of age at her first marriage. A researcher claims that for women in California, this estimate is too low. Surveying 213 newlywed women in California gave a mean of 25.4 years with a standard deviation of 2.3 years. Using a 95% level of confidence, determine if the data supports the researchers claim.
Ho: $\mu = 25$ years       Ha: $\mu > 25$ years

$n = 213$

mean assumed $\mu_0 = 25$ years

$\alpha = 5\% = $ Type I = significance level – English: 
(reject the survey statement of 25 years as an error)

$\beta = 95\% = $ Type II = confidence level – English: 
(fail to reject the survey statement of 25 years as an error)

experimental value $\bar{x} = 25.4$ years
Calculations

\[ s_\bar{x} = \frac{2.3}{\sqrt{213}} = 0.1575 \]

\[ Z_c = \text{invNorm}(0.95) \approx 1.65 \text{ table} \]

\[ Z_\bar{x} = \frac{25.4 - 25}{0.1575} = 2.538 \]
Solution

$SE = 0.1575$

$\mu_{\bar{x}} = 25$

$Z_{c} = 1.645$

$\bar{x} = 25.4$

Locate experimental value =
conclusion statistically

Reject Ho, accept Ha

conclusion in English

reject the survey statement of 25 years
accept the researchers claim of > 25 years

\[ p\text{-value} = 0.00155/6 \]

\[ \text{normalcdf}(25.4, B, 25, 0.1575) \]
\[ 0.005547679 \]

\[ \text{normalcdf}(2.538, B) \]
\[ 0.0055744214 \]

\[ p\text{-value interpretation} - \text{smallest } \alpha \text{ necessary to reverse the conclusion ; results are very significant.} \]
A recent study showed that the average number of children for women in Europe is 1.48. A global watch group claims that German women have an average fertility rate which is different from the rest of Europe. To test their claim, they surveyed 128 German women and found that an average fertility rate of 1.39 children with a standard deviation of 0.84. Does this data support the claim made by the global watch group at the 90% level of confidence?
Ho: $\mu = 1.48$ rate  \hspace{1cm} Ha: $\mu \neq 1.48$ rate

$n = 128$

mean assumed $\mu = 1.48$

$\alpha = 10\% = \text{Type I} = \text{significance level} - \text{English: (reject the study claim 1.48 fertility rate as an error)}$

$\beta = 90\% = \text{Type II} = \text{confidence level} - \text{English: (fail to reject the study claim 1.48 fertility rate as an error)}$

experimental value $\bar{x} = 1.39$
\[
\sigma_{\bar{x}} = \frac{0.84}{\sqrt{128}} = 0.0742
\]

\[
Z_c = \begin{cases} \text{invNorm}(0.95) \\ 1.644853626 \\ \text{invNorm}(0.05) \\ -1.644853626 \end{cases} \approx \pm 1.65 \ table
\]

\[
Z_{\bar{x}} = \frac{1.39 - 1.48}{0.0742} = -1.212
\]
SE = 0.0742

Locate experimental value = $x = 1.39 \text{rate}$

$Z_c = -1.645$

$\mu_\text{x} = 1.48 \text{rate}$

$Z_c = 1.645$

$Z_{\text{x}} = -1.212$

$2\text{TT} \neq$
conclusion statistically

**Fail to Reject Ho**

conclusion in English

Fail to disprove the study claim of a 1.48 fertility rate.

$p$-value = 0.1123

$p$-value interpretation – **smallest $\alpha$ necessary to reverse the conclusion**; results are **not significant**.
A manufacturer must test that his bolts are 2 cm long when they come off the assembly line. He must recalibrate his machines if the bolts are too long or too short. After sampling 100 bolts, he calculates the sample mean to be 1.9 cm and the standard deviation to be 0.5 cm. Assuming a level of confidence of 0.95, is there evidence to show that the manufacturer needs to recalibrate machines?
Ho: $\mu = 2 \text{ cm}$  Ha: $\mu \neq 2 \text{ cm}$  
n = 100  

mean assumed $\mu_x = 2 \text{ cm}$  

$\alpha = 5\% = \text{Type I = significance level}$  

-English: (reject the claim of 2 cm as an error)  

$\beta = 95\% = \text{Type II = confidence level}$  

-English: (fail to reject the claim of 2 cm as an error)  

Experimental value $\bar{x} = 1.9 \text{ cm}$
Calculations

\[
SE = \sigma \frac{x}{\sqrt{100}} = \frac{0.5}{\sqrt{100}} = 0.05
\]

\[
Z_c = \frac{1.9 - 2}{0.05} = -2
\]

\[
\approx \pm 1.96 \text{ table}
\]
Solution

\[ \mu_x = 2 \text{cm} \]
\[ Z_c = 1.96 \]
\[ Z_{\bar{x}} = -1.96 \]
\[ \bar{x} = 1.9 \text{cm} \]

\[ SE = 0.05 \]

Locate experimental value = \[ \bar{x} = 1.9 \text{cm} \]
conclusion statistically

Reject \( H_0 \), accept \( H_a \)

conclusion in English

Manufactured sample is not within specifications. Retooling is necessary.

\( p \)-value = 0.0228

\( p \)-value interpretation – smallest \( \alpha \) necessary to reverse the conclusion; results are significant.
CNN Money reports that the average cost of a speeding ticket, including court costs, was $150 in 2002. A local police department claims that this amount has increased. To test their claim, they collect data from 160 drivers that have been fined in the last year and find that they paid an average of $154 per ticket with a standard deviation of $17.54. Is there evidence to support the police departments claim at the 0.01 level of significance?
Ho: $\mu = $150  Ha: $\mu > $150

n = 160

mean assumed $\mu_x = $150

$\alpha = 1\% = \text{Type I = significance level} - \text{English:} \ (\text{reject the CNN statement of $150 as an error})$

$\beta = 99\% = \text{Type II = confidence level} - \text{English:} \ (\text{fail to reject the CNN statement of $150 as an error})$

experimental value $\bar{x} = $154
Calculations

\[
\sigma \approx \frac{17.54}{\sqrt{160}} = 1.386
\]

\[
Z_c = \frac{\text{invNorm}(0.99)}{2.326347877} \approx 2.33 \text{ table}
\]

\[
Z \approx \frac{154 - 150}{1.3866} = 2.88
\]
SE = 1.3866

\[ \mu = 150 \]
\[ Z = 2.88 \]
\[ Z_c = 2.33 \]
\[ x = 154 \]

Locate experimental value =
conclusion statistically

Reject Ho, accept Ha

conclusion in English

reject the CNN statement of $150
accept the claim of > $150

\[ p\text{-value} = 0.00198 \]

\[ \text{normalcdf}(154, B, 150, 1.3866) = 0.0019586638 \]
\[ \text{normalcdf}(2.88, B) = 0.0019884417 \]

\[ p\text{-value interpretation} - \text{smallest } \alpha \text{ necessary to reverse the conclusion} ; \]
\[ \text{results are very significant}. \]