Nuclear Stability

What is the nuclear stability? Nuclear stability means that nucleus is stable meaning that it does not spontaneously emit any kind of radioactivity (radiation). On the other hand, if the nucleus is unstable (not stable), it has the tendency of emitting some kind of radiation, i.e., it is radioactive. Therefore the radioactivity is associated with unstable nucleus:

Stable nucleus – non-radioactive
Unstable nucleus – radioactive

Keep in mind that less stable means more radioactive and more stable means less radioactive.

We want to know why there is a radioactivity. What makes the nucleus a stable one? There are no concrete theories to explain this, but there are only general observations based on the available stable isotopes. It appears that neutron to proton (n/p) ratio is the dominant factor in nuclear stability. This ratio is close to 1 for atoms of elements with low atomic number and increases as the atomic number increases. Then how do we predict the nuclear stability? One of the simplest ways of predicting the nuclear stability is based on whether nucleus contains odd/even number of protons and neutrons:

Protons	**Neutrons**	**Number of Stable Nuclides**	**Stability**
Odd	Odd	4	least stable
Odd	Even	50	↓
Even	Odd	57	
Even	Even	168	most stable

- Nuclides containing odd numbers of both protons and neutrons are the least stable means more radioactive.
- Nuclides containing even numbers of both protons and neutrons are most stable means less radioactive.
- Nuclides contain odd numbers of protons and even numbers of neutrons are less stable than nuclides containing even numbers of protons and odd numbers of neutrons.

In general, nuclear stability is greater for nuclides containing even numbers of protons and neutrons or both.

Example

Based on the even-odd rule presented above, predict which one would you expect to be radioactive in each pair?

(a) $^{16}_{8}O$ and $^{17}_{8}O$

(b) $^{35}_{17}Cl$ and $^{36}_{17}Cl$

(c) $^{20}_{10}Ne$ and $^{17}_{10}Ne$

(d) $^{40}_{20}Ca$ and $^{45}_{20}Ca$

(e) $^{195}_{80}Hg$ and $^{196}_{80}Hg$

Answer

(a) The $^{16}_{8}O$ contains 8 protons and 8 neutrons (even-even) and the $^{17}_{8}O$ contains 8 protons and 9 neutrons (even-odd). Therefore, $^{17}_{8}O$ is radioactive.

(b) The $^{35}_{17}Cl$ has 17 protons and 18 neutrons (odd-even) and the $^{36}_{17}Cl$ has 17 protons and 19 neutrons (odd-odd). Hence, $^{36}_{17}Cl$ is radioactive.

(c) The $^{20}_{10}Ne$ contains 10 protons and 10 neutrons (even-even) and the $^{17}_{10}Ne$ contains 10 protons and 7 neutrons (even-odd). Therefore, $^{17}_{10}Ne$ is radioactive.

(d) The $^{40}_{20}Ca$ has even-even situation and $^{45}_{20}Ca$ has even-odd situation. Thus, $^{45}_{20}Ca$ is radioactive.

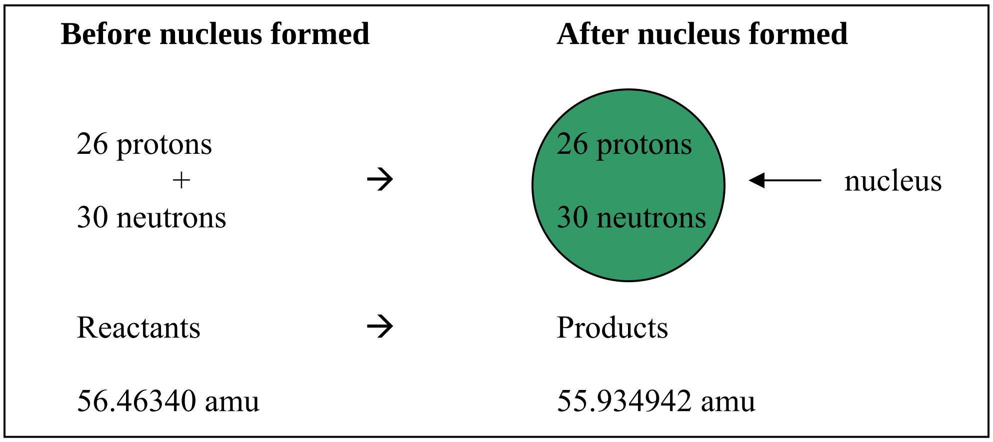
(d) The $^{195}_{80}Hg$ has even number of protons and odd number of neutrons and the $^{196}_{80}Hg$ has even number of protons and even number of neutrons. Therefore, $^{195}_{80}Hg$ is radioactive.

Nuclear Binding Energy

The nuclear binding energy is an energy required to break up a nucleus into its components protons and neutrons. In essence, it is a quantitative measure of the nuclear stability. The concept of nuclear binding energy is based on Einstein's famous equation, $E = mc^2$, where E is the energy, m is the mass and c is the velocity of light, and according to which the energy and mass are inter-convertible.

Nucleus contains mainly two particles – protons and neutrons- in addition to many other elementary particles. Thus, the mass of the nucleus is primarily comes from the masses of protons and neutrons. But the experiments have shown that the sum of the masses of protons and neutrons is always greater than experimentally determined nuclear mass. Why is it so? The answer to this question lies in the way the nature creates nucleus. When nature creates nucleus, it takes protons and neutrons and binds them together and puts them in a tiny space called nucleus. In order to bind protons and neutrons together, some energy is needed, which is taken out of the masses of protons and neutrons. It means that nature is very smart, it does not spend any of its own energy rather it converts some of the masses of protons and neutrons into an energy and utilizes that energy to bind the protons and neutrons within the nucleus. If we know how much mass (known as mass defect) is utilized, we can convert it into binding energy using the Einstein's equation. Let us see how this is done.

Consider ${}^{56}_{26}Fe$ that has an atomic mass of 55.934942 amu (experimental) that is created using 26 protons and 30 neutrons:



This nucleus has 26 protons and 30 neutrons. Knowing the mass of proton and neutron, we can calculate the total mass of the nucleus, i.e., total mass of 26 protons and 30 neutrons. The mass of proton (${}^{1}_{1}H$) is 1.007825 amu and that of neutron (${}^{1}_{0}n$) is 1.008665 amu. Thus

Mass of 26 protons = 26 x 1.007825 = 26.20345 amu
Mass of 30 neutrons = 30 x 1.008665 = 30.25995 amu

Total mass of 26 protons and 30 neutrons is

26.20345 amu + 30.25995 amu = 56.46340 amu

This mass is larger than 55.934942 amu (the experimentally determined mass) by 0.52846 amu.

The difference between experimental mass of the atom and the sum of the masses of its protons, neutrons, and electrons is known as **mass defect** (Δm), which is calculated as

$$\begin{aligned}\Delta m &= \text{mass of products} - \text{mass of reactants}\\ &= \text{experimental mass of an atom} - \text{calculated mass of an atom}\\ &= 55.934942\ \text{amu} - 56.46340 = -0.52846\ \text{amu}\end{aligned}$$

Note that Δm is a negative quantity. As a consequence, the calculated energy will also be negative because the formation of ${}^{56}Fe$ from 26 protons and 30 neutrons is an exothermic reaction meaning that the energy is released to the surrounding. Also note that Δm does not include the mass of electron, as electron mass is much smaller than mass of either proton or neutron and hence it can safely be omitted.

This mass defect can be further transformed into energy using Einstein's equation in the following form:

$$\Delta E = \Delta m \times c^2$$

In this equation, ΔE is the change in energy in joule, Δm is the mass defect in amu, and c is the velocity of light that is equal to 3.0×10^8 m/s. Substituting these values into above equation and converting all the units to joules gives the energy in proper units (J), which is of course little bit tedious. To make things simpler, one can directly convert the mass defect into energy using the following conversation factor (if you are interested, see the following box for derivation of this relationship).

$$1\text{amu} = 1.4945 \times 10^{-10} \text{ J}$$

Let Δm = 1 amu. Then

$$\Delta E = \Delta m \times c^2 = 1 \text{ amu} \times (3.0 \times 10^8 \text{ m/s})^2$$
$$= 9 \times 10^{16} \text{ amu m}^2/\text{s}^2$$

We further utilize the following relations to convert amu m^2/s^2 into joules.

$$1 \text{ kg} = 6.022 \times 10^{26} \text{ amu}$$
and
$$1 \text{ J} = 1 \text{ kg m}^2/\text{s}^2$$

Then $$\Delta E = 9 \times 10^{16} \text{ amu m}^2/\text{s}^2 \times (1\text{kg} / 6.022 \times 10^{26} \text{ amu}) \times (1 \text{ J} / 1\text{kg m}^2/\text{s}^2)$$

$$= 1.4945 \times 10^{-10} \text{ J}$$

That is, $1 \text{ amu} = 1.4945 \times 10^{-10}$ J

Therefore,

$$\Delta E = \Delta m \times 1.4945 \times 10^{-10} \text{ J/amu} = -0.528458 \text{ amu} \times 1.4945 \times 10^{-10} \text{ J/amu}$$

$$= -7.8978 \times 10^{-11} \text{ J/ nucleus}$$

This is the amount of energy released when one iron-56 nucleus is created from 26 protons and 30 neutrons. Therefore, the nuclear binding energy for this nucleus is 7.8978×10^{-11} J, which is also the amount of energy required to decompose this nucleus into 26 protons and 30 neutrons. The above calculated energy is per nucleus. The energy released for the formation of 1 mole of iron nuclei is calculated by multiplying the above energy with Avogadro's number.

$$\Delta E = -7.8978 \times 10^{-11} \text{ J} \times 6.022 \times 10^{23} \text{ /mol} = -4.7560 \times 10^{13} \text{ J /mol}$$
$$= -4.7560 \times 10^{10} \text{ kJ /mol}$$

Therefore, the nuclear binding energy for 1 mole of iron-56 is 4.7560×10^{10} kJ (this is about 48 billion), which is a tremendous amount of energy, considering the energy of ordinary chemical reactions that are usually in the order of 200 kJ. How tremendous this energy is? Well, it will heat up about 40 million gallons of water from room temperature to boiling point. Can you imagine just about 56 g of iron-56 has the ability to heat up about 40 million gallons of water? Pretty impressive, is it not?

Binding energies are usually reported per nucleon to facilitate the comparison between various binding energies, which is calculated using the following formula.

$$nuclear\ binding\ energy\ per\ nucleon = \frac{nuclear\ binding\ energy}{number\ of\ nucleons}$$

For the iron-56 nucleus, we have

$$nuclear\ binding\ energy\ per\ nucleon = \frac{7.8978x10^{-11}\ j}{56} = 1.4070x10^{-12}\ j\ /\ nucleon$$

Click on

Hands on Practice to Calculate Nuclear Binding Energy